Modern Asset Pricing and Project Evaluation in the Energy Industry

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1. Introduction

A key task of any commercial organisation is the choice of its asset configuration. This choice cannot be made without the generation and evaluation of alternatives. Asset valuation is usually a key aspect of the evaluation process.

For several decades in the energy industry, the most common form of asset valuation for these purposes has been a style of Discounted Cash-Flow (DCF) analysis, which, in this paper, we call the Standard DCF approach. However, over the past five years in particular, an increasing number of organisations in the upstream petroleum and electrical generation industries, among others, have been experimenting with the use of another approach. The most common term now for this is Real Option Valuation (ROV), although, for reasons that are made clear in Section 1.5, we prefer the more general term: Modern Asset Pricing (MAP).[1]

At this point, the future role of MAP is not clear,[2] and there is much fundamental work that remains to be done in developing MAP technology. However, there is enough activity and interest in the use of MAP methods, particularly in the energy industry, that it is appropriate to undertake a selective review of what is known publicly, and what remains to be done, on this topic.

This review builds on a special issue of The Energy Journal on the topic of “The Potential for Use of Modern Asset Pricing Methods for Upstream Petroleum Project Evaluation”, guest edited by one of us (Laughton 1998a).

In this introductory section, we address the following questions, with reference, where needed, to The Energy Journal issue.

1) What is wrong with Standard DCF? (It is costly to introduce new valuation techniques, so there has to be a reason for doing so.)
2) How does MAP overcome some of the deficiencies of Standard DCF?
3) What ideas are behind the MAP approach to asset valuation?

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[1] MAP has also been called Black-Scholes or Black-Scholes-Merton analysis (after the originators of the field: Black and Scholes 1973, and Merton 1973), derivative asset valuation (because it deals in part with the valuation of assets that derive their value from the value of other assets), and contingent claims analysis (e.g., Merton 1977, Mason and Merton 1985). Predecessors of some aspects of MAP have also been called adjusted net present value analysis (Myers 1974) and valuation by components (Lessard 1979).

[2] The Economist (2000) recently reported that, according to Bainand Co., 46% of a sample of firms that had experimented with ROV had abandoned their experiments. Is the glass half full or half empty? How well were these firms positioned for this type of experiment? Did they receive the best possible advice on how to proceed? If not, would the adoption rate have been higher with the wider availability of better advice? Will the adoption rate be different in the future?

4) How are MAP analyses done?
5) Why we are reviewing MAP rather than restricting our attention to Real Option Valuation (ROV), which is the focus of almost all of the attention and writing in this area?

We then list some issues that need to be addressed about the use of MAP in the energy industry.

The rest of the paper is a review of publicly reported progress in addressing the issues raised in the Introduction. Our intent is to provide some assistance to a group of readers, who have made some initial attempts to use MAP methods, and who want to know more about the technical aspects of implementing MAP analyses as they proceed to the next step in doing so. If the reader is a novice with MAP or ROV, we would suggest that he or she refer to the appropriate parts of The Energy Journal (Laughton 1998a) as needed.[3] More experienced ROV analysts may find this review useful as well, but more selectively, and possibly because we present a view of ROV that is not exactly the same as their own.

To keep the paper manageable, we focus on the upstream petroleum industry, with only a few references to electricity generation. Furthermore, we discuss the valuation of production assets exclusively, although similar issues arise in the valuation and management of long-term sales agreements.[4]

Sections 2 and 3 examine two modelling issues that arise in MAP analyses:

1) the construction of the scenario tree and the policy set to be used in the analysis; and
2) the determination of state prices for states on the scenario tree.[5]

Section 4 deals with computational issues. Section 5 describes some publicly reported applications. Section 6 mentions two empirical studies. Section 7 concludes.[6]

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[3] Readers should note that the vertical scale in Figures 2a, 2b, 3a and 3b of Salahor (1998) is wrong. The divisions on the vertical axis should be increments of $0.70/mcf rather than $0.50/mcf. There are also minor errors in Tables 3a and 3b of Salahor (1998). These and other errata are available from the guest editor of the issue.

[4] Tufano (1996) gives a good non-technical introduction to some of the issues involved in the pricing and management of sales agreements, using the gas marketing business of Enron as an example. Mello and Parsons (1995) discuss one example, Metallgesellschaft, of how things can go wrong.

[5] The terms “scenario tree” and “state price” are defined in Section 1.3, and “policy set” in Section 2.8.

We have found, for our own purposes, some characteristics of publications in this field that signal their usefulness for learning more about how to apply MAP in the energy industry. We list some of these characteristics in an Appendix.

While, in this paper, we deal primarily with modelling and computational issues in performing a MAP analysis, there are important - indeed, much more important - organisational issues surrounding the process by which MAP might be brought into use within a corporation or government department or agency.

This process should go ahead in stages. Each step must result in a big enough change to make the effort worthwhile. However, it must be small enough change to be manageable in an uncertain political environment.

How these changes are made is very important. Formal valuation is only a part of the asset structuring process. However, it does provide part of the language for the discussion surrounding these decisions (Laughton 1988, Luehrman 2000). Moreover, any formal valuation method will use imperfect and incomplete representations of the world, and must be calibrated by the experience of the decision-makers in using them. For these two reasons, well-managed organisations are conservative in the changes they make in this area of their activities, and rightly so. They move slowly, and changes made must respect the legacy of what has gone before, which, in this case, is the Standard DCF approach to valuation.[7]

The choice of whether and, if so, when and how to introduce MAP into any organisation will depend on the potential evolution of the costs and benefits of doing so. The overall trend is for a decrease in costs, as more disaggregated financial market data is created, better MAP technology is developed, and more basic training in this field occurs. Overall, the level of benefits will increase with the increasing complexity of the world business environment. However, the benefits and costs, and their evolution, for any given organisation will also depend, to some extent, on the organisation itself, its history, and the details of its business. We return to this with some brief comments in the Conclusion in Section 7.

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[6] Some readers, who are experienced with ROV or who want to look at results before examining technique, may wish to skim over the rest of Sections 1 to 4, look at the rest of the paper, and then return to the rest Sections 1 to 4.

[7] There are some other useful references on these organisational issues. Laughton (1998d) and Faiz (2000) discuss aspects of the process for initial introduction of MAP into an upstream petroleum organisation. Claeys and Walkup (1999) give a nice discussion of some of the issues involved in organising a given ROV analysis within the same type of company.

Finally, before we continue, we should give one caution. The premises on which MAP is based are only approximately valid in reality, so care must be taken in the use of MAP techniques in situations where the differences between these approximations and the "real world" are important: There are inherent limits to what MAP can do well. This is pointed out in Section 1.3. Moreover, there are additional limits to what can be done with MAP now, while it is still being developed. These limits will change as time goes on. This is the subject of some discussion throughout the paper.

1.1. The Standard DCF Approach and its Limitations

The Standard calculation of DCF project values is done in three steps.

1) The analyst calculates, for each scenario to be considered, the project cash-flow that will occur at each time in that scenario. This is done by inserting the determinants of the cash-flow, such as the output price, for that scenario into a model of the project structure.

2) The project cash-flows for each scenario are discounted, typically using a discount rate that is common to a wide class, if not all, of the projects to be considered by the organisation undertaking the analysis.

3) The discounted cash-flows are added to form the project "value" in each scenario.

The set of "scenario values" forms one of the quantitative inputs into the decision-making process.

In some versions of the Standard DCF evaluation method, a probability distribution for the scenarios is constructed, and the quantitative aspects of the evaluation are extended by a statistical analysis of the scenario values. For example, the expectation of the values (called the "expected net present value" or ENPV) is frequently used as a measure of the overall "return" of the project. Measures of the "risk" in the project include the standard deviation in the scenario values, the difference between the maximum and minimum values, and the probability that the value is less than some threshold (usually zero).

There are at least four problems with this.

First, Standard DCF methods can induce significant and systematic biases into the financial analyses that are part of the evaluation process. The constant discounting used to produce values for large classes of projects is based on the false premises that:

1) the risks in different projects are of the same magnitude; and
2) these risks are resolved at a constant rate over time.

Moreover, the process used almost always presumes that all project decisions are made at the time of the analysis.

Among other things, this usually produces biases against long-term or strategic decision-making by discounting the future excessively,[8] and by undervaluing the ability of project managers to respond to future contingencies.[9] Decision-makers can attempt to compensate for these biases, if they are aware of them. However, other things being equal, it would be better to use valuation techniques that avoid the bias in the first place.

Second, and very important, the focus of the analysis on "now or never" alternatives does not encourage the generation of project alternatives that incorporate explicitly opportunities for future management flexibility.

Third, Standard DCF analysis depends critically on the choice of a project discount or hurdle rate, yet many organisations do not understand the issues behind this choice. There are good reasons for this, as these issues can be very complex. It would be preferable to use valuation methods that avoid as much of this complexity as possible.

Finally, Standard DCF methods lead managers to consider risk in ad hoc ways through some combination of their choice of a discount or hurdle rate, and their opinion of the spread in valuation results across "sensitivity" scenarios. It would be better to have an integrated approach to risk and its effects on value.

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[8] Salahor (1998) gives an example of this phenomenon. The undervaluation of long-term assets was flagged in the 1980s by many commentators as a cause of a problematic short-term focus of many private commercial organisations (Hayes and Garvin 1982, MacCallum 1987, Dertouzos et al. 1989). It was suggested frequently that financial evaluation be ignored in favour of "strategic" analysis. MAP shows that the problem lay not with financial analysis itself, but with incomplete and biased forms of financial analysis. One theme of MAP is to bring financial discipline to strategic analysis and strategic reality to financial analysis. Myers (1984, 1987) made this point at the time (although his technical advice is now dated). Amram and Kulatilaka (1999a, 1999b) have emphasized it again recently.

[9] Managerial options include the timing of different stages of the project cycle: exploration and delineation (in the case of natural resource development), initial feasibility and design studies, initial development, production and decommissioning. Managers may also have choices about the speed at which pre-development work is done, about initial technology and capacity, phased development of subsequent production facilities, and the technology and capacity of each phase, partial or complete temporary shutdown or partial decommissioning, and mid-phase expansions. There may also be marketing flexibility, including risk-sharing options on energy prices in energy sales contracts.

1.2. How Does MAP Mitigate the Deficiencies of Standard DCF?

Modern Asset Pricing (MAP) instructs the analyst to perform the equivalent of the first two steps in the Standard DCF valuation process in the reverse order.[10]

1) The analyst discounts the uncertain project cash-flow determinants (e.g., the output prices), using discounting structures appropriate for each determinant, and, if needed, constructs a distribution around these discounted determinants.

2) These discounted cash-flow determinants (or the distribution around them, if need be) are filtered through the project structure to find the different parts of the project value.[11]

3) These parts of value are added to form the total asset or project value.

Where relevant, this project structure includes the possibilities of how managers may intervene in the future, on a contingent basis, to influence asset value.

Because the discounting is done at the level of the cash-flow determinants, there is no need to determine or use a project discount rate. However, if desired, one can be calculated as a consequence of the valuation (Salahor 1998, Bradley 1998). It arises jointly from the discounting of the project determinants and from the project structure. This demonstrates that the MAP valuation process incorporates the effects of project structure on value, not only through its effects on the forecast of project cash-flows, but also through its effects on their risk.

Also, if there is flexibility in the future management of the project, the MAP process allows the analyst to search for an optimal policy, or a constrained optimal policy, for managing that flexibility, using the project value calculated concurrently as a guide.

How does this process mitigate the problems posed by Standard DCF methods?

First, if done properly, it removes the bias induced by the inappropriate use of a single hurdle rate for projects with different patterns of risk. It also allows analysts to include the value of future flexibility in their evaluations.

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[10] More details of how these calculations are done are given in Sections 1.4 and 4, and in Salahor (1998), Bradley (1998) and Laughton (1998c).

[11] In Section 1.4, we show how the value can be split in different ways depending on what is most convenient or simplest under the circumstances.

Second, it gives more encouragement to the generation of a wider variety of project alternatives for consideration, including those where future management flexibility is an issue.

Third, discounting individual project determinants, as MAP does, involves many fewer considerations than directly discounting project cash-flows. To use an example, a barrel of oil to be received 10 years from now is a much simpler asset than a producing field, let alone an undeveloped field, a group of fields, an exploration prospect or an exploration play. MAP asks project analysts to discount the price of future barrels of oil, while Standard DCF methods demand that they discount directly the cash-flows of the more complex assets.

Fourth, MAP methods integrate the analysis of risk and value by determining project value as a property of all possible scenarios considered as a group, as opposed to determining a "value" for each scenario in the group of scenarios, and using this group of "values" in the evaluation process.

One worry often expressed about MAP methods is that, if each project were to have its own discount rate, senior management would have less control of the project evaluation process, and project proponents would have a greater opportunity to “fiddle” the analysis to their own ends. Salahor (1998) explains why this is not the case, and why, on the contrary, there is less opportunity for “fiddling” the analysis if MAP methods are used.

1.3. What Ideas Are Behind the MAP Approach to Asset Valuation?

Four ideas are linked together to form the basis of MAP. Each should be familiar to many readers. What is new is the detailed exposition and combination of them in the MAP framework. We outline the ideas, and, in Section 1.4, describe how MAP implements them. The introduction and the first three papers in Laughton (1998a) expand upon the short description given of these ideas here.

The first idea is that a project (or, more generally, any asset) can be valued by considering the incremental cash-flow that it uses and generates. Cash-flow is a commodity and should be valued according to those of its characteristics that are important to the people who trade in it. For cash-flow, these are timing and risk. [12]

[12] DCF methods recognise this in the use of discount rates that combine a risk-free rate (valuation of time) and a risk premium (valuation of risk) to discount cash-flow amounts in the determination of asset values

Time is a relatively simple concept. Its effects on value are priced by using the term structure of prices for claims to risk-free cash-flow. These prices may be expressed in terms of risk-free interest rates.

Risk is more complex. Salahor (1998) gives a textbook discussion of what risk means to investors who have access to a diversified financial market. He distinguishes between “priced” risks, which cause non-zero risk discounting, and “unpriced” risks, which do not. [13]

Priced risks are correlated with the determinants of welfare of the marginal diversified investor. They are also called “nondiverisfiable”, “systemic”, “market” or “macroeoconomic” risks. An example would be oil price risk.

Unpriced risks are not correlated in this way. They are also called “diversifiable”, “nonsystemic”, “local”, “private” or “project-specific” risks. An example would be the underlying geological risk in a small petroleum exploration prospect.

The second idea is that financial markets have low enough transaction barriers and costs that, to a good approximation, trading quickly drives the price of two different assets to be the same if those assets have the same cash-flow characteristics. This is the principle of "value consistency" (sometimes called, the "no-arbitrage" principle, the "Modigliani-Miller" approximation,[14] or, more graphically, the "no-free-lunch" principle).[15]

One special form of consistency is the principle of "value additivity", which, though simple, is very powerful. It allows us to take the cash-flows that would stem from ownership of an asset, break them up into convenient pieces for valuation, and then add the values of the pieces to get the value of the asset. Following from the principle of value additivity is the result that, if one cash-flow is the multiple of another in all situations, the value of the claim to the first

[13] A better term might be “zero-priced” risks, instead of “unpriced” risks, but we shall use the more common term.

[14] The seminal papers on value consistency are Modigliani and Miller (1958) and Miller and Modigliani (1961).

[15] Transaction barriers and costs in financial markets can be important in certain types of situations. This is one key factor in determining the limits of the usefulness of MAP. For example, if it were not for transaction costs and barriers, markets would be complete, the details of corporate financing and risk management would be irrelevant, and financial intermediaries would be unimportant. Therefore care must be taken in using MAP in situations where any of incompleteness of markets, corporate financing or risk management, or financial intermediation is at issue.

cash-flow is the same multiple of the value of the claim to the second. We stretch the principle of value additivity to include this result.[16]

Another special form of consistency is “rollover valuation”, which allows us to determine the value of a multi-period asset by replicating its cash-flow through a process of trading period by period in (or “rolling over”) single-period assets.[17]

These first two ideas are all that is needed to value a wide class of assets, where asset cash-flows are linear combinations of variables (such as commodity prices) for which forward prices exist or can be determined (Salahor 1998). We elaborate on this in Section 1.4.1.

The third idea is that, if necessary, the future can and should be represented by a tree-like structure of possible scenarios. The paths through the tree (and thus the end states) represent different scenarios. The intermediate nodes (also called "states") on the tree represent sets of information that differentiate amongst groups of possible remaining scenarios. The branching from any of these states represents the arrival of new information that differentiates further among the remaining groups of scenarios.[18]

Any asset can be valued using these three ideas, if the (possibly contingent) policy for managing the asset can be specified before the valuation is done. Cash-flow is split up into the parts occurring at each state (instead of merely at each time). The claim to each of these parts may be valued separately under conditions, and using methods, described in Section 1.4.2. The parts of value for each state can then be added across the states to find the project value.

If the management of future flexibility must be analysed within the context of the valuation, we need to use the fourth idea: that policies for managing the asset should be framed in terms of actions to be undertaken at various states on the scenario tree. A search should be made over possible policies to find those that add the most value.

[16] Value additivity is behind the additive form of the DCF valuation formula for projects with cash-flows that occur at different times.

[17] Rollover valuation lies behind the general theory of compound interest that is the basis of the discounting model for valuation of individual cash-flows.

[18] Those readers who are familiar with the so-called “binomial lattice” approach to option valuation (Cox, Ross and Rubinstein 1979) should understand that this notion of branching is slightly different from the branching used in the binomial approach and much more general. We return to this in Section 4.3.2.

This last idea is the basis of Decision Tree Analysis (DTA), with which many people in the petroleum industry are familiar. Indeed, one of the canonical applications of decision tree analysis is the management of the exploration of a petroleum prospect.[19]

1.4. How Are MAP Analyses Done?

The choice of technique for doing a MAP analysis depends on the complexity of the situation to be analysed. The most complex situations are those where the future management of the asset must be analysed within the context of its valuation. This is the case of Real Option Valuation (ROV). The mechanics used in ROV can be used in all situations, at least in principle. However, the analysis can be significantly simplified if there is no management flexibility, or if the management policy to be used can be specified before the valuation is done. Furthermore, there is one especially simple set of situations to which we turn first.

1.4.1. Linear Cash-flow Models

The simplest type of MAP calculations occur when the cash-flows to be valued depend linearly on underlying variables for which forward prices exist or can be calculated.[20] In this case, as we noted in Section 1.3, only the first two ideas behind MAP are needed to structure the calculation. The example used in Salahor (1998) is a gas field with known production and cost profile in the face of uncertain gas prices. The cash-flow at each time is modelled to have the form:

output * gas price – cost. (1)

Each of these cash-flows may be split into two parts: the revenue and the cost. Each part may be valued separately, and the values added using the principle of value additivity.[21] Salahor (1998) shows that the resulting value of

[19] Good references for DTA include Raiffa (1968) or more recently Haimes (1998).

[20] A “forward contract” is an agreement between two parties in which one party agrees to deliver a unit of commodity to the other at a specified time, “called the settlement date”, and price, called the “forward price” for that time.

[21] This approach stems from early work on adjusted net present value (Myers 1974), which attempted to calculate the value of a project in two parts: the project considered on an all-equity basis and an adjustment stemming from financing considerations, which would typically have a different risk. Lessard (1979) elaborated on this by being more radical about splitting the project cash-flow into components, along the lines of this example. Both of these references are dated in their technique.

the claim to each cash-flow is:

(output * gas forward price – cost) * unit risk-free cash value. (2)

The unit risk-free cash value discounts the cash-flow for time. The cost is risk-free, so it need not be discounted for risk. The revenue is discounted for risk by using the gas forward price, which, by an application of value consistency, can be shown to be gas price discounted for risk (Salahor 1998).

This type of analysis generalises to situations where the cash-flow depends in a linear way on more than one uncertain underlying variable. The value of the claim to the cash-flow is calculated by inserting the appropriate forward price for each underlying variable into the cash-flow model and multiplying the result by the unit value of risk-free cash.

In situations where the forward prices do not exist (because the relevant forward contracts, or their equivalent, are not traded in a liquid market), a model of the forward prices must be created. Salahor (1998) and Baker et al (1998) address this issue.[22]

1.4.2. Other Situations Without Real Options

In situations where project cash-flows do not have a linear structure, we cannot use the specific valuation methods just outlined.

One important cause of cash-flow nonlinearity is future flexibility in project management. There are special complications if the best policy for managing flexibility can only be determined within of the context of the valuation. We turn to this in Section 1.4.3, where we examine Real Option Valuation (ROV).

Here we show how to calculate value in situations where the project cash-flows are nonlinear for other reasons. These can include:

1) nonlinear fiscal terms;
2) political risk;
3) nonlinear models relating price and cost;
4) the combination of input and output uncertainties with price uncertainties; or
5) the existence of managerial flexibility where the management policy is determined before the valuation is done.

[22] In particular, Salahor (1998) estimates forward prices by discounting, for risk, the expectation of the variable in question. He uses prices and amounts of risk, and shows how models like the Capital Asset Pricing Model (CAPM) are actually models of prices of risk.

The key is to bring into play the third idea ("use scenario trees to represent the future"). The basic notion is that once a scenario tree is in place, the states on the tree can be used to define a partition of project cash-flows for valuation. Then the principle of value additivity is invoked to add the value arising from the cash-flow in each state.

To implement this method, we need to know, for each state on the tree, how to value a claim to cash-flow received only in that state, and then how to add these values.

To do this, the "unit state cash-flow" for each state is defined as the cash-flow that has a unit amount if and when the state is realised, and otherwise is zero. The claim to this cash-flow is called the “state asset” for that state, and the value of the state asset is the “state price” (Bradley 1998).

To value a pattern of cash-flow, we use the principle of value additvity, by taking the product of the state price and the cash-flow in each state, and summing this product over all the relevant states.

Bradley (1998) discusses how financial data can be used in the determination of state prices, in some simple situations where the scenario tree is constructed so that portfolios of assets with known prices can be dynamically traded to replicate the consequences of holding each of the relevant state assets. Baker et al. (1998) elaborate on his discussion.

For now, we should note that each state price is composed of three factors:

1) a time discount factor equal to the unit price of risk-fee cash-flow for that time;
2) the true probability that the state will occur; and
3) a risk adjustment if priced risks are involved.

The product of the true probability and the risk-adjustment form a risk-adjusted probability for the state. Roughly speaking, this will be higher than the true probability in states where investor welfare is otherwise low, and lower where it is otherwise high. If no priced risks are involved, the true and risk-adjusted probabilities are the same.

Finally, Bradley (1998) shows the relationship between MAP and expected net present value (ENPV) analysis. In doing so, he demonstrates how MAP captures some essential effects of risk on value that ENPV, by using an ad hoc model of risk adjustment, misses.

1.4.3. Real Options

Real Option Valuation (ROV) complicates the asset valuation method just outlined in Section 1.4.2 by requiring a search for the best policies to manage the asset in the future. Moreover, the possible dependence of parts of the scenario tree on future management actions becomes an important consideration when more than one possible policy is to be analysed.[23]

In principle, a valuation can be done for each contending policy as if it were the only one available. The best strategies can then be found by comparing the resulting values. If the trial valuations are costly to compute (as is usually the case), a smart search strategy will try to make the number of trial valuations required as small as possible.

The search method most commonly used in the literature has been dynamic programming. The valuation begins at each end state on the scenario tree, where the value is the terminal cash-flow. The algorithm then works back through the tree calculating, at each non-terminal state, the action, or set of actions, at that state that give the highest value, and this value itself. The value generated by an action at a state is the sum of the cash-flow in that state from that action and the value that comes from the asset value in the states that immediately follow in the scenario tree. This follow-on value is calculated using state pricing methods.[24]

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[23] The resolution of some uncertainties is not under the control of the management. These include:

1) uncertainty about the prices that managers face when they sell into, or buy from, a market in which they are small participants; and
2) uncertainty about the direction and rate of overall technological progress of relevance to the industry.

We call these “exogenous” uncertainties. The part of the scenario tree that deals with the resolution of exogenous uncertainties is self-contained, and we refer to it as the “exogenous scenario tree”.

Managers may influence or control the resolution of some other uncertainties. They are typically uncertainties about the state of the project itself rather than the world in which it is undertaken. They may include geological uncertainties about mineral deposits, or uncertainties about the efficiency and effectiveness in the particular project of the use of different combinations of technology. We call these “endogenous” uncertainties

[24] For each follow-on state, the value of the asset in that state is multiplied by the state price for that follow-on state as registered in the state being analysed. These products are then summed to determine the follow-on value. A detailed description of this is given in Laughton (1998c) and Section 4.3.3 of this paper.

1.5. Why a Review of MAP and Not ROV Alone?

Real Option Valuation (ROV) has become a “hot” topic in the corporate finance world: the subject of several articles in the business press, promoted by several major consulting firms, and examined at a great many conferences.[25] It is also the topic of almost all Modern Asset Pricing (MAP) literature and MAP applications.[26]

The exact scope of ROV is the subject of some ambiguity. However, it usually refers to the application of MAP in situations where asset managers have some future flexibility in asset management. For the purposes of this paper, we are a bit more specific, and refer to ROV as the application of MAP in situations where the asset valuation and the determination of the policy for future management of the asset are done jointly.[27] In these situations, the fourth idea (“use decision tree analysis (DTA)”) is applied to the analysis. Conversely, ROV may be considered, in our usage, to be DTA where the decision payoffs are valued using MAP methods.[28]

There are four reasons why we are not restricting our attention to ROV as we have defined it.

1) There are several types of important insights to be obtained from the use of MAP evaluation methods in situations where flexibility, and thus, ROV, is not an issue (e.g., Salahor 1998, Bradley 1998, Jacoby and Laughton 1992, Lund 1992, Majd and Myers 1986).

2) Situations where flexibility is an issue are typically harder to comprehend and analyse than those where it is not. A step-by-step approach to learning the theory and practice of MAP that deals with

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[25] Business press articles include The Economist (1991, 1994, 1999), Amram and Kulatilaka (1999b), Amram, Kulatilaka and Henderson (1999), Copeland and Keenan (1998a, 1998b), Coy (1999), Dixit and Pindyck (1995), Jackson (1997), Leslie and Michaels (1997), Leuhrman (1998a, 1998b), Mintz (1999, 2000), Moore (2000), Paul-Choudhury (1999), Popper (1999), Rappaport (2000), Stackpole (2000), Tufano (1996), and Valdmanis (1999). Major consulting firms that have real options practices include McKinsey, PriceWaterhouseCoopers and Ernst and Young. Recent Conferences, where real options papers have been presented, include the Canadian Institute of Mining and Metallurgy AGMs, valuation workshops for the Society of Petroleum Engineers, various meetings of the International Association for Energy Economics, and several conferences sponsored by the International Quality and Productivity Centre, EuroForum, and The Real Options Group.

[26] Some of this literature is in monograph form. This includes Amram and Kulatilaka (1999a), Dixit and Pindyck (1994), and Trigeorgis (1996). There are also books of papers such as Lund and Øksendal (1991), Trigeorgis (1995), Micalizzi and Trigeorgis (1999), Brennan and Trigeorgis (2000), and Trigeorgis (2000).

[27] This is the definition of ROV used in Laughton (1998a).

[28] This point is also made by Smith and McCardle (1999).

increasingly complicated situations is possible if the broader approach is taken. Complicated ROV applications can then be considered once the valuation basics are understood in less complicated situations.

3) In Sections 4.1 and 4.2, we demonstrate that MAP Applications, that do not involve ROV, are close in form to Standard DCF analyses. Therefore, a staged introduction of MAP would deal first with situations where the parallels to the Standard DCF approach are strongest.[29]

4) The MAP and DTA aspects of ROV are logically separate. As we have noted, MAP can be used in situations where DTA is not required. Moreover, DTA can use some other form of payoff evaluation (e.g., a utility evaluation or a constant discount rate DCF valuation). Having a broader focus allows an approach that deals separately with the issues that arise from the use of MAP, and those that arise from the use of DTA. This makes it easier to identify the source of different evaluation effects.

1.6. The Issues

There are several modelling or computational issues that arise from the use of MAP in the energy industry.

1) The modelling of the scenario tree must be detailed enough to give an accurate representation of those aspects of the uncertain future that are essential for the asset valuation being done. Conversely, it must be parsimonious enough to allow for a parameterisation of the state prices and to permit the sum over states that is a part of the valuation. Finally, it must be tuned enough to our knowledge of financial market prices to allow the state price parameters to be calculated with a sufficient degree of confidence.

2) If flexibility is an issue, the set of strategies considered must be broad and detailed enough to capture the essence of the important choices that managers will face. It must also be small enough to keep the scenario tree, and the search for the best policies, manageable.

3) There is typically not enough financial market information to determine the needed state prices without some assumptions being made about their structure. Care must be made with these assumptions. Their

[29] In fact, a DCF analysis done according to the textbooks (e.g., Brealey et al. 1986, which advises that the expected cash-flows should be discounted using a discount rate with an appropriate risk premium) is merely a special case of MAP analysis. The forward contract, if it existed, for the project cash-flow at each future time would have a forward price, discounted from cash-flow expectation using the risk premium in the DCF discount rate.

consequences must be tested rigorously for their consistency and reasonableness.

4) The states on the scenario tree are frequently modelled to have a continuous structure. If this is so, the sum over the states is an integral, which must be summed numerically. Moreover, if flexibility is at issue, the search for optimal management strategies can be computationally very intensive. How these computational issues are managed is frequently a major determinant of the feasibility of a desired analysis. The more efficient this management, the less stringent are the modelling constraints mentioned above.

These considerations have been, and continue to be, the subjects of active research. They are also, and will continue to be, the source of (sometimes subtle) tradeoffs in well-done applications, and possible sources of error in poorly performed work. Their consideration provides the framework of the review that follows.

2. The Scenario Tree and the Policy Set

In a MAP project valuation, the scenario tree provides the backbone for describing the uncertainty in the project cash-flows. Decisions about its structure are key to any MAP analysis involving nonlinear cash-flows. They are particularly important in the case of an ROV analysis.

The choice of the variables that will be supported on the scenario tree is the key modelling decision. The more variables there are, the larger the tree and the more difficult the calculations. Of particular importance are those variables that involve priced risks. The determination of the risk adjustments in state prices can impose tight modelling constraints, and large informational and computational costs, on the analysis, as we show in Section 4.3.

2.1 Early Work

The key insight of Black, Scholes and Merton, in their original work on MAP (Black and Scholes 1973, Merton 1973), was their state pricing method for the valuation of cash-flow claims. They focus on situations where the cash-flow amount is determined by the concurrent price of a single “underlying” asset, such as a corporate equity. [30]

[30] Merton (1977) gives a good technical explanation of the Black-Scholes-Merton method of analysis.

First, they find that there are significant analytical simplifications if the underlying asset price is approximated to be a continuous variable, and if trading in assets is presumed to occur continuously.

Second, they show that, under certain conditions, a complete understanding of how the future might look moving forward from any given state in the tree depends only on the then current underlying asset value, and not on the history leading up to that state.[31] This means that any analysis done locally in time can depend on the one-dimensional continuum of possibilities for the then current underlying asset value.

Third, using the principles of value consistency, they show that, under these same conditions, the state cash-flows for these recombined underlying asset price states can be replicated by continuous trading in the underlying asset and claims to risk-free cash. The sequence of portfolios formed by the process are called "replicating portfolios", which are traded according to a "dynamic replication strategy".

The relevant conditions, used by Black, Scholes and Merton, are that:

1) the term structure of prices to risk-free cash must be known with certainty;
2) the dividends from the equity and the uncertainty in its price must depend only on time and that price; and
3) the asset price uncertainty must be compact enough so that continuous replication is feasible.

Their first application was the valuation of European stock call and put options.[32] They were able to determine relatively simple formulae (called the Black-Scholes formulae) for these prices[33] in terms of observable or calculable parameters, namely:

1) the contractual parameters of the option;
2) the stock price;
3) the risk-free interest rate;
4) the amount and timing of any dividend to be paid before the exercise time; and
5) the proportional uncertainty (or "volatility") in the stock price.

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[31] This is called the "markov" property of the tree.

[32] These are the right to buy (for a call) or sell (for a put) a given corporate equity at a given future time (the "exercise time") for a fixed price (the "exercise price"). A stock is a corporate equity. An option is called a European option if the right can be exercised at one given time.

[33] These formulae are the result of the state price determination through their replication process and the state-pricing sum of the option cash-flows.

In these formulae, the option price depends on two combinations of these parameters. This dependence can be presented in tabular form.[34]

They also extended their analysis to the case of American options.[35] The formulae no longer apply necessarily, but the same replication methods may still be used to set up the valuation.

These initial applications and their extensions influenced much of the early work in MAP applied to real assets. Researchers looked for situations where they could draw analogies to the Black-Scholes-Merton analysis of equity options. They then applied the Black-Scholes-Merton method in an analogous way.

Representative examples of this work include:

1) the analysis of the option to delay initiation of a project (McDonald and Siegel 1986);
2) an empirical analysis of the value of offshore oil field leases in the USA (Paddock, Siegel and Smith 1988); and
3) the analysis of the option for premature abandonment of a project (Myers and Majd 1984, 1990, McDonald and Siegel 1985).

All of these use the claim to the operating cash-flows of the underlying project as the asset analogous to the corporate equity in the stock option analysis. All analyse "timing options" by determining a boundary, possibly dependent on time, between two sets in the underlying asset price. In one of these sets, on one side of the boundary, the managers should act to change the state of the project, and in the other set, on the other side of the boundary, they would wait, if allowed to do so, or walk away from the change, if not.

There are two problems with this type of analysis.

First, the claim to the operating cash-flows of a real project has a complicated dynamics that is difficult to model transparently over time scales required. In particular, the analogy of the flow of stock dividends in the stock option analysis is the operating cash-flow itself. The Black-Scholes-Merton condition that the cash-flow at any time depends only on time and the concurrent value of the claim to the whole stream of cash-flows is very restrictive.[36] In a sense,

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[34] See, for example, Appendix Tables 6 and 7 in Brealey et al. (1986). Equivalent tables may be found in many good introductory finance texts.

[35] An American option gives the owner the right to buy (for a call) or sell (for a put) at any time up to a given date.

[36] For example, if the cash-flow is proportional to the value (e.g., Majd and Pindyck 1987), it cannot be negative.

it puts the cart before the horse. We should model the value of the claim to the cash-flow stream in terms of the cash-flows themselves, if we can, and not the other way around. Furthermore, the condition that the uncertainty in the value of the operating cash-flows depends only on that value is also very restrictive.

Second, many projects have imbedded in them more than one timing option and options that are not timing options at all, but options among different project designs or operations. If there are more options than a single timing option, there may be no unique definition of the claim to the operating cash-flows to be used as the underlying asset.

It has been the experience of two of us (Laughton, Samis) that attempts to shoehorn real projects into this very restrictive framework have proved frustrating for several of those organisations that have tried to use it.

This brings up a major difference between the application of MAP to support decision-making about real assets and the use of MAP to support trading in financial assets. Financial assets are defined contractually and their complexity can be controlled, while real assets are presented to managers by the real world, and their complexity is determined primarily by that world rather than by the managers themselves. This means that it is important to have a broad and flexible modelling environment in a MAP facility set up to value real assets.

2.2. The Brennan-Schwartz Mine

After this early work, there was a shift to scenario trees where the scenario tree variables are the determinants of the project cash-flows, whether they be asset values or not. As a result, the scenarios in the scenario tree are more like the scenarios that would be used in a Standard DCF analysis. This type of scenario tree also provides a much broader modelling environment within which to work.

This shift had already begun within the valuation by components framework of Lessard (1979). However, the key step was the use by Brennan and Schwartz (1985) of the copper price as a scenario tree variable in their ROV analysis of copper mines that can be temporarily closed.[37] They use the copper price in much the same way that Black and Scholes (1973) and Merton (1973) use the underlying asset price in their analysis. Like the underlying asset price in the stock option analyses, it is a continuous variable

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[37] This work was sponsored in part by the Canadian Ministry of Finance, which was interested in the effects of government policy on closure decisions.

that evolves continuously. In their model, a complete understanding of how the future of the copper market might look moving forward from any given state in the tree depends only on the then current copper price, and not on the history leading up to that state. In doing this, they build on the work of Cox, Ingersoll and Ross (1985a,b), who use the short-term interest rate as a scenario tree variable in a valuation of long-term bonds and bond options.

Brennan and Schwartz also use the remaining amount of copper in the mine, and the state of the mine (open or closed), as scenario tree variables. The mine cash-flows and the actions of managers depend on these variables as well as the copper price. Although they do not have any independent uncertainty in the Brennan-Schwartz model, these variables do depend on the prior history of copper prices, which would have influenced prior opening and closing decisions. Instead of using the whole price history to index states on the tree, Brennan and Schwartz use these "auxiliary" scenario tree variables to represent those aspects of the history that have an influence on future cash-flows.

Brennan and Schwartz are very parsimonious in their modelling of the uncertainty that influences a mine. They have no geological or technological uncertainty, and no independent cost uncertainty. They have only one output price.[38] The relevant dynamics of that one output price is influenced in their model only by itself, with no independent uncertainty in the interest rate, in the amount of uncertainty in the price, or in the equivalent of the dividend yield from holding the output.[39] They did this to keep the dynamic programming search for optimal opening and closing policies feasible. However, they are extravagant in one sense. They allow cash-flow to occur continuously and decisions about opening or closing the mine to take place at any time.

2.3. Continuous-Time Information and Discrete-Time Cash-flows and Decisions

In most DCF analyses, cash-flows are modelled to occur at discrete (usually annual) time intervals. In most decision tree analyses, decisions and payoffs are also restricted to occur at discrete times. In reality, cash-flows occur on an almost continuous basis, with some large flows at discrete intervals or random discrete times. Most regular planning decisions occur within the context of a periodic planning cycle, while some extraordinary decisions occur at random times.

[38] Most metal mines have more than one metal output.

[39] This is called the "convenience yield", and is discussed in more detail in Section 3.1.2.

As we show in Section 4.3.3, MAP analyses can be simplified significantly if cash-flows and decisions are modelled to occur at discrete intervals as in typical DCF and DTA analyses. If this is done, only a finite number of the characteristics of each scenario on the scenario tree comes into play, as opposed to a continuously infinite set. There are other simplifications that occur in the analysis of timing options and their effects, to which we return in Section 4.3.3.

The errors caused by this restriction on cash-flows and decisions can be tested by examining the changes in the results that occur as the time interval is decreased. To our knowledge, there has been no systematic set of tests done to determine the typical size of the errors involved.

2.4. Individual Commodity Prices

The copper price model used by Brennan and Schwartz (1985) has its limitations, which we explore in Section 3.1.4. In particular, it is not consistent with the existence of forces that tend to lead commodity markets into long-term equilibrium.[40] Salahor (1998) gives a good discussion of this, and Laughton and Jacoby (1993), Salahor (1998), Bradley (1998) and Smith and McCardle (1999) describe a model that deals with this issue while maintaining the same scenario tree structure that occurs in the Brennan-Schwartz (1985) model. Unfortunately, for reasons mentioned in Laughton (1998d), this model has its own problems.

In a modelling framework that uses price forecasts as the scenario tree variables, Laughton (1988) addresses this problem, by having a long-term factor and a separate short-term factor in the uncertain forecast changes. Gibson and Schwartz (1990) and Schwartz (1997) incorporate the "convenience yield"[41] as an additional tree variable in a model that is shown by Schwartz and Smith (2000) to be equivalent to a model where the price is the product of a long-term factor and a short-term factor. Further details of these models are given in Section 3.1.4.2.

[40] Mello and Parsons (1995) show how ignoring the existence of long-term equilibrium forces can have disastrous effects in the management of long-term hedged marketing agreements.

[41] The concept of "convenience yield" is described in more detail in Section 3.1.2. It is the difference between the expected return required in the financial markets to hold the commodity and its expected rate of price change. It represents the net benefit (or "convenience") of holding the commodity itself rather than some future claim to it. The analogy in the dividend yield for a stock. Both are called "rate-of-return shortfalls".

2.5. Other Variables with Priced Risks

There has been little work reported in the literature on scenario trees for more than one variable that involves priced risks. Schwartz (1997) examines an extension of his two-factor single commodity price model to include interest rate uncertainty, using the short-term rate as a scenario variable. Routledge, Seppi and Spatt (1999) examine the generation of electricity from natural gas using a model with both commodity prices.

However, there has been no publicly reported joint modelling for MAP purposes of oil and natural gas prices, although many upstream petroleum projects produce both products. Should the two be modelled separately or should the gas price be modelled as a deviation from the energy-equivalent oil price?

There is little publicly reported work on the representation of different types of cost uncertainty and their links with output price uncertainty. How should costs be split up for analysis? In the petroleum sector, are drilling costs, for example, influenced, possibly with lags, by movement and levels in the output prices, through rent and quasi-rent effects? Do they have significant uncertainties from other commodity prices, or from general real price indices?

There are special issues for those parts of the energy industry where one form of marketed energy is transformed into another (e.g., heavy oil upgrading, oil refining, electricity generation from fossil fuels).[42] In these sectors, the operating cash-flows of a production asset depend directly on the spread between the spot prices of input and output commodities. For example, the operating cash-flows of a fossil-fuel electricity generation project will depend on the "spark spread", which is the difference in market price between a megawatt of electricity and the heat equivalent of the fossil fuel needed to produce power (e.g., oil, natural gas or coal). The equivalent construct in oil refineries is the "crack spread" between the price of the refinery products (e.g., heating oil and gasoline) and the price of the crude oil used.

There are two approaches to modelling the cash flows from operations when a commodity price spread is involved. The first is directly to model the spread itself, while the second is to model the constituents and then subtract one variable from the other (e.g., Routledge, Seppi and Spatt 1999). The first approach offers parsimony, which is important if the model is already complicated. The second approach allows one to capture stylistic facts regarding cross commodity correlations, and more accurately to model operations (e.g., a nonlinear production technology).

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[42] We could also include the transport and distribution of energy in this category where the transformation is a shift in location.

Finally, there may be reasons to go deeper into the systematic determinants of the cash-flows than the input and output prices that we have thus far considered. The next step would be to go back to the determinants of supply and demand. Some work has been done in this direction, which we discuss in Section 3.1.3. Moreover, a look at a broader framework is necessary for dealing with political risk in situations where the behaviour of the relevant governments is driven by considerations broader than the prices involved in the base project evaluation.

2.6. Unpriced Risks

The scenario tree variables we have considered thus far involve priced risks, as defined in Section 1.3. Unpriced risks (i.e., risks for which the price of risk is zero) can also be important. Recall that these include local, project-specific risks that do not influence the risk discounting in financial market prices because their effect is not noticeable in the determinants of the welfare of the marginal well-diversified investor. Moreover, the timing of their resolution is frequently influenced by management action, which makes them "endogenous" uncertainties. Finally, we do not need to determine a Black-Scholes-Merton replication process to find their contribution to the risk-adjustments imbedded in state prices, because we already know the effect (zero) by other means. This last point gives the analyst more freedom in modelling their uncertainty.

The unpriced risks most commonly considered in the literature are the geological risks in petroleum and mineral resource extraction. This includes overall uncertainties about the amounts of petroleum in place, as well as uncertainties that can influence possibilities for the shape of the production profile.

Frimpong (1992) examines a discrete lognormal process for resolving reserve uncertainty through staged delineation, using the expected reserve as the scenario tree variable. Amram and Kulatilaka (1999a) also set up the structure of a model based on the sequential resolution of forecast reserve size (but did not present any analysis of it). Chorn and Carr (1997) model a continuous time resolution of reserve size, resolved through exploration and production. Pickles and Smith (1993) consider situations with dry hole uncertainty. Laughton (1998c) and Smit (1997) each analyse a simple model with dry-hole risk and discrete wet-hole uncertainty in reserve size resolved sequentially through exploration and delineation.[43]. Smith and McCardle (1999) also consider reserve uncertainty in their examples, resolved through exploration, development and production, and residual production profile

[43] Laughton (1998c) is a simplification of an unpublished report (Laughton 1994) that allows for the gradual resolution of wet-hole reserve-size uncertainty through exploration, delineation and development.

uncertainty resolved through production. Hanson and Laughton (1994) look at the uncertainty in waterflood response for secondary recovery of heavy oil, and how it may be partially resolved through a pilot waterflood.

There has been less work on unpriced cost uncertainty. Laughton, Frimpong and Whiting (1993) consider a simple taxonomy of single-stage project-level research programmes, including programmes that eliminate a discrete unpriced uncertainty in project costs. Smith and McCardle (1999) consider unpriced cost uncertainty that is resolved through the life cycle of a petroleum project. Pindyck (1993) examines a stylised form of unpriced cost uncertainty resolution in a continuous time framework.

More work should be done in this area to explore appropriate tree structures for unpriced risks, using the flexibility afforded by having no need to manage a replicating portfolio determination of state-price risk adjustments.

2.7. Auxilliary Scenario Tree Variables

Auxiliary variables, like the remaining reserves and the mine status in Brennan and Schwartz (1985), are not subject to any direct uncertainty, but inherit uncertainty from the behaviour of prior variables. Other examples of auxiliary variables include tax status variables such as a tax loss carryforward balance.

The modeller can ease considerably the computational burden that results from the presence of such variables, if cash-flow and decisions are modelled to occur at discrete intervals. Under these circumstances, auxiliary variables have a discrete-time dynamics, and discrete realisations if their evolution depends only on a discrete set of decision alternatives. We deal with the implications of this in Section 4.3.3.

2.8. The Policy Set

An asset management policy specifies a single management action for each state on the scenario tree. This may be called the “action function” of the policy. The “policy set” for a given asset valuation consists of all possible policies under consideration. The valuation will be a Real Option Valuation (ROV), according to our definition, if, prior to the asset valuation, there is at least one future state where there is more than one action that may be taken, with the result that the policy set includes more than one possible policy.

The number of possible actions allowed by the policy set categorises the decision environment at any scenario tree state. Broadly, the policy set may allow one action, a finite number of actions (greater than one), or an infinite number.[44] Most applications restrict themselves to models where, at each state, there are only a finite number of alternatives, possibly by making a discrete approximation to a continuum of possibilities.[45] We discuss only policy sets that satisfy this restriction.

2.8.1. A Simpler Labelling of Some Policy Sets

It is always possible to label a policy by its action function. However, in many situations, it is useful to represent policies by specifying the boundaries between the regions in scenario tree where different actions are taken.

For example, in their analysis of the timing option to develop a project, McDonald and Siegel (1986) use the value of the developed project to label states at any given time in the scenario tree. In any state, there are at most two possible actions: “begin the project” and “do not begin the project”. It can be shown that the best policy for managing the timing option, at any given time, must have the following form: In states with a value above a critical value, the project is begun (if it has not already begun); otherwise, it is not begun. As a result, each policy under consideration may be labelled by this critical value considered as a function of time, or, in other words, by the boundary between the two sets of states where the different actions are taken under the policy. This is a much simpler labelling than the action function.

This may be generalised to any situation where:

1) there are, globally on the scenario tree, a finite number of possible actions to be taken;
2) states in the scenario tree are labelled so that they can grouped into regions with easily described boundaries; and
3) the policies that need to be considered prescribe each action to be taken in a group of states that can be easily described by its boundaries.

2.8.2. A Partial Taxonomy of Policy Sets

Many projects can be treated as having a life cycle with a series of possible phases, and transitions, optional or not, between these phases.

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[44] An infinite number of alternatives may exist if, for example, management can choose from among a continuum of production levels, or a decision is being made about the direction, duration and cost of a research programme or development plan.

[45] One exception is Sagi (2000), who considers the choice of cut-off grade and production in a mining project, from a continuum of possibilities.

The early work mentioned in Section 2.1 focusses on situations where there is a single “transition option” that separates two possible phases in the project life cycle. These models allowed management to choose between two possible actions: “continue with the first phase”, or “shift to the second”. For example, McDonald and Siegel (1986) analyse an investment option where the two phases are the pre-investment phase and the project operation phase. Some of this work, including that of McDonald and Siegel (1986) incorporates a “timing option” that permits the manager to choose, at each time in a (possibly continuous) set of times, whether to make this transition. An auxiliary variable, indicating whether the transition has been made previously and the time the transition occurred, can obviate the need to label scenario tree states with a more detailed description of the scenario history to which they belong. This was done implicitly in the early work.

Brennan and Schwartz (1985) significantly broadens the types of policy sets that can be considered by allowing multiple timing options with more than one possible transition alternative (e.g. either abandoning or reopening a closed mine). This creates the situation where managers in some project states have more than two actions from which to choose. Their model is carefully designed to prevent an explosion in the scenario tree, and they begin the formal use of auxiliary variables to represent the effects of uncertain prior actions.

In some situations, transitions between phases in the project life cycle, whether they are optional or not, may also involve some “design options”, such as technology or capacity choice. In states where these design options occur, there are frequently more than two possible actions to consider. This is always the case if the transition itself is optional.

There may also be “operational options” within a phase of the project life cycle (e.g., choice of production level or sequence).

Finally, some of the options may be “research options” where the project managers have a choice of when and/or how to resolve some project-level uncertainty (e.g., through mineral exploration, resource delineation, or feasibility or design studies). As we have already noted, the presence of these options makes the scenario tree dependent on the policy under consideration.[46]

[46] The use of auxiliary variables that depend on past management actions means that the labelling of states on the tree may be influenced by the policy under consideration, but the tree itself is not changed by this form of labelling.

2.8.3. Decisions at Discrete Times

Restricting cash-flows and decisions to a discrete set of times makes the project -specific aspects of the scenario tree look like those in an ordinary DTA analysis. Nevertheless there are constraints, from the size of the scenario tree, on the level of policy detail that can be considered practically. These constraints are particularly important for modelling those actions that can resolve project-level uncertainty, and in limiting the number of alternatives that can be considered in design and operating options. The constraints may be computational constraints, data constraints, or constraints on the amount of information that can be presented to decision-makers. It is not clear whether these constraints are more or less ornerous (and, if so, by how much) in ROV than in other types of DTA. This would be a useful issue to explore.

3. The Determination of State Prices

Once the scenario tree is in place, the most crucial choices to be made in a MAP analysis are those that determine the state prices for the states on the tree. Different models, even those based on the same financial market data, can sometimes lead to vastly different valuations and "optimal" asset management policies. [47] Therefore, the analyst should undertake a thorough testing of as many different features as possible of the models being considered.

Following the work of Brennan and Schwartz (1985), and because of the data available from commodity futures and forward markets, [48] most of the literature deals with state prices for traded commodity price states. After a discussion of the issues raised in such models, we shall turn briefly to modelling state prices involving priced risks when there is no clean forward market for such risks. We close with a brief comment about unpriced risks.

[47] See, for example, Schwartz (1997), who compares several different models fitted to the same data sets.

[48] "Future contracts" are similar to forward contracts. However, to avoid default, cash is transferred at the end of each day up to the settlement date, rather than only at the settlement date itself. The "futures price" is the competitive settlement price for this daily transfer. The cash change in the daily futures prices is subtracted from an account (called a "margin" account) of the buyer of the contract and added to a margin account of the seller. This activity is called "marking to market". To enhance liquidity, future contracts are traded on exchanges, and exist for a few standardised settlement dates.

3.1. Traded Commodity Price Models

3.1.1. The Importance of Forward and Futures Prices

State prices for states on a commodity price scenario tree must be consistent with financial market prices influenced by those commodity prices.

Consider first the case of a commodity that is traded directly in financial markets and offers no benefits to its owner from being held physically. If the uncertainty in its value depends only on that value and time, and if this uncertainty is compact enough to allow replication, then the state prices can be determined using the methods of Black and Scholes (1973) and Merton 1993, 1997). A strategy must be found for trading the commodity and risk-free assets over time to replicate the state cash-flow. According to the principle of value consistency, the state price must be the value of the initial portfolio needed to begin the trading strategy.

If the commodity does not satisfy these criteria, a "bridging" derivative asset can be used in its place, if such a derivative exists.[49] Brennan and Schwartz (1985) show that future and forward contracts come close to filling this niche, if the value from holding the asset depends only on the price imbedded in the contract.

Forward prices provide important information about state prices (indeed the only information needed for valuations with linear cash-flow models, as we note in Section 1.4.1). Each forward price is the expectation of the corresponding future spot price discounted for risk. As such, it is also the expectation of that spot price with respect to the risk-adjusted probability distribution imbedded in the scenario state prices. Futures prices are important because they are very close to forward prices and are more easily determined in liquid financial markets.[50]

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[49] A bridging asset embodies the future value of the physical commodity and yet is treated as a financial asset by all market participants.

[50] Cox, Ingersoll and Ross (1981) and Richard and Sundaresan (1981) derive the relationship between futures and forward prices. If compounded interest rates are uncorrelated with spot prices, then futures and forward prices coincide. In particular this is true under the approximation of known (i.e., non-stochastic, but possibly time-dependent) interest rates. Schwartz (1997) and Pindyck (1994) examine crude oil and heating oil futures, respectively, and find that the inferred difference between forwards and futures, due to interest rate correlations, would be negligible in the context of a MAP application. We know of no similar tests published for natural gas or electricity.

Other features of actual futures and spot markets demand that care must be taken when using observed futures prices in econometric analyses to determine state price parameters.

For example, settlement is usually not at a given fixed time. One of the parties usually has an option to choose the delivery time within a set period. Electricity futures, for example, may

3.1.2. Some Stylized Facts About Forward and Futures Prices

In an arbitrage-free market, a portfolio long on a futures contract with maturity, t + Δ, and short on a future with maturity t will have a time t risk-adjusted expected return of:

$$R(t, \Delta) = (r_t + s_t - \eta_t)\,\Delta \tag{3}$$

where r_t is the time t risk free rate of interest, s_t is the rate of storage costs paid between t and $t + \Delta$, and η_t corresponds to a rate of benefits of ownership. The last term arises because, unlike financial assets, physical assets have some immediate benefit of consumption. In particular, a consumer is generally not indifferent between the physical asset and its cash equivalent. η_t is an aggregate measure of this, and is commonly termed the "gross convenience yield". We shall refer to $\eta_t - s_t \equiv \delta_t$ as the "net convenience yield" or simply the "convenience yield".

The most important regularities evident in commodity futures are the presence of "backwardation" and "contango". Strong backwardation in the forward term structure corresponds to negative slope near settlement; weak backwardation corresponds to a slope less than r_t near settlement. Contango is distinguished by a slope greater than r_t near settlement. In terms of the parameters introduced in Eqn. (3), weak backwardation occurs when the convenience yield is positive, and strong backwardation occurs when it exceeds the short risk-free interest rate. Backwardation is usually attributed to the economic benefits associated with having the commodity on hand, as opposed to a deferred right of ownership. Between 1984 and 1992, for example, crude oil, was in strong (weak) backwardation 77% (94%) of the time (Litzenberger and Rabinowitz 1995). Contango, on the other hand, occurs when there are higher benefits to deferring consumption (e.g., heating oil in summer).

Other important aspects of futures/forward prices include the following.

1) Seasonal variation in energy futures prices is exhibited by heating oil, gasoline, natural gas and electricity. Seasonality also appears often in the convenience yield (Pindyck 1994). This is naturally attributed to

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be settled at any time in the month of delivery. There may also be other delivery options (e.g., location of delivery, details of quality). Fama and French (1987) note that the underlying spot market is usually not very liquid. Indeed, the nearest futures price is often quoted as the spot price (Schwartz 1997). Long-term futures (greater than 12 months) are thinly traded and may therefore fail to incorporate all information in their prices (Gabillon 1995). Idiosyncrasies in trading contracts, particularly at the end of the month, are often reflected in the futures price (Serletis and Hulleman 1994).

weather and can give rise to predictable regimes of backwardation and contango (i.e., oscillations in the futures price term structure).

2) Large spikes in commodity prices due to unexpected weather, political events or other major news are not rare (Hilliard and Reis 1998). Electricity, due to its non-storability, is particularly prone to sudden and violent jumps in price (Kaminski 1997). These find expression in the forward term structure.

3) Crude oil, heating oil, natural gas and, likely, electricity price series are strongly heteroskedastic[51] (Duffie and Gray 1995). With oil, the price volatility is positively correlated with the convenience yield (Litzenberger and Rabinowitz 1995).

4) Oil products confirm the Kaldor-Working hypothesis that aggregate inventories are negatively correlated with the convenience yield (Brennan 1991, Gabillon 1995).

5) Relative volatility of futures prices tends to decrease with increasing maturity. This is known as the Samuelson (1965) effect and signals reversion to some long-term equilibrium trend. For evidence in energy commodities, see Serletis and Hulleman (1994) and Bessembinder, Coughenour, Seguin and Smoller (1995).

6) There is some evidence for correlation between macroeconomic risk factors and returns from a futures position (Bailey and Chan 1993). It is controversial, however, to conclude empirically that futures prices incorporate non-zero risk premia, as this conclusion seems to depend on the period through which a futures position is held (Deaves and Krinski 1995).

7) Exchange rates can be highly correlated with commodity prices, particularly when producing countries are less developed (Gilbert 1991).[52] Changes in currency exchange rates can affect demand for primary commodities entailing new equilibrium prices.

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[51] A time series is heteroskedastic if unexpected shocks (residuals) are not identically distributed.

[52] Political developments and uncertainty clearly plays an important role in both determining exchange rates as well as commodity prices - sometimes in tandem and sometimes separately.

3.1.3. Theoretical Insights into the Forward Term Structure

A number of theories have been proposed to explain different empirical facts. These can be loosely classified into three categories. The first, originating with Keynes (1930), attempts to frame the persistent backwardation in most commodities as resulting from a risk premium offered by hedgers to insurers. The second theoretical category, promoted by Kaldor (1939) and Working (1949), conjectures that inventory and the potential for stockouts are responsible for much of the observed regularities. The last category, inspired by the ROV literature, conjectures that backwardation is required in equilibrium in order to entice producers to extract resources. In what follows, we review these main strands of the literature in light of the stylized facts mentioned in Section 3.1.2

3.1.3.1. Theories of Risk and Return

A futures price curve that tends to be in strong backwardation implies that any long position[53] involves an expected flow of cash each day as the contract matures and is marked to market. For that reason, Keynes (1930) suggests that insurers, who would take a long futures position thereby guaranteeing hedgers a fixed price for their goods, demand a premium. Cootner (1960) extends this to argue that hedgers were sometimes consumers, as opposed to producers, in which case insurers would demand a premium for short positions - hence contango.

More recently, economic theorists, such as Richard and Sudaresan (1981), Breeden (1980) and Grauer and Litzenberger (1979), have formalised the Keynes and Cootner intuition in consumption-based general equilibrium models. As with any financial asset, the risk premium associated with a futures position depends on the correlation between the price of the commodity and aggregate marginal consumption. More specifically, commodities that are positively correlated with aggregate marginal consumption will be in backwardation (commodities that move in tandem with the business cycle are hedged by producers); commodities that are negatively correlated with aggregate marginal consumption will be in contango (commodities that move counter to the business cycle are hedged by consumers).

As mentioned above, empirical support for the details of these theories is mixed (Deaves and Krinski 1995). Moreover, the pure risk-return approach ignores the effects of inventory, for which there is empirical support if the commodity is storable. Indeed, Williams (1987) questions whether the

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[53] The party that receives the goods at maturity of the futures contract is said to take a "long" position. The counterparty is then said to be "short".

existence of futures markets is based primarily on the demand for risk sharing services. He presents an equilibrium model in which risk-neutral producers and consumers hedge as a result of incurring higher spot market transaction costs than speculators.

3.1.3.2. Theories of Storage

The "theory of storage", promoted by Kaldor (1939) and Working (1949), singles out inventory as the main driving force relating spot prices to futures prices for storable commodities. If inventories could be negative, backwardation in futures markets would be ruled out by the no-arbitrage principle. To see this, imagine that spot prices are higher than the nearby futures price, and consider a portfolio consisting of a short position in inventory and an offsetting position that is long on the nearby futures contract. The "borrowed" inventory is sold for a price higher than the locked-in price that must be paid to close the position and settle the inventory debt. The pocketed difference is arbitrage profit. Since physical inventories cannot be sold short, the lesson is that backwardation can appear when inventories are low.[54] This also suggests that inventory is negatively correlated with convenience yield.

These ideas are formalised by Routledge, Seppi and Spatt (1999) (following Deaton and Laroque (1992, 1996) and Chambers and Bailey (1996)) in a rational expectations equilibrium model where the probability of a stockout causes positive convenience yields.[55] They demonstrate the practicality of their model with an example based on a discrete binary-branching tree, which can be used to price other derivatives and real options. However, as pointed out by Miltersen and Schwartz (1998), the marginal convenience yield and spot price will only be correlated when the commodity is in short supply.

3.1.3.3. Other Theoretical Considerations

Litzenberger and Rabinowitz (1995) use an ROV approach to equilibrium forward pricing and point out that in equilibrium, producers require a positive convenience yield to extract exhaustible resources. A producer has the option to extract up to some capacity each period. Such a timing option is

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[54] Contango is, however, bounded by an arbitrage argument. Buying the commodity cheap today and locking in a high price for it tomorrow should not yield a return greater than the riskless rate plus the cost of storage.

[55] Heinkel, Howe and Hughes (1990) present a related model in which physical inventory has an embedded option over a long futures position whenever demand shocks are not persistent. In their model, the holder of inventory can benefit from sudden short-term shortages, which do not affect longer maturity futures. The Gulf War crisis was an example.

analogous to an American call and will be exercised only if benefits are lost by deferring production. The lost benefit is embodied in backwardation or positive convenience yield, which Litzenberger and Rabinowitz show has the structure of a put option. In their framework contango can occur if producers cannot easily shut down production (e.g., the hysteresis effect in Brennan and Schwartz 1985).

Some nice recent work by Carlson, Khokher and Titman (2000) set up a demand and supply equilibrium model for a non-renewable resource market, where the demand function is stochastic and reverts to a long-term mean, and there is an unlimited substitute, the cost of which follows a geometric brownian motion. [56] In the presence of a cost for increasing overall production, which is linear in that increase, they show the resource price exhibits reversion in their model, because of the cost of increasing production. One of the consequences of this reversion is that the forward prices exhibit the Samuelson effect. Moreover, forward prices can either be in backwardation or in contango depending on whether the demand is above or below its long-term mean.

The theoretical approaches summarized here emphasize different intuition about the underlying economics of commodity markets. Researchers are only beginning to integrate the ideas and so a generally acceptable equilibrium theory is future work. Some of the current general equilibrium approaches, such as in Richard and Sundaresan (1981), do not even develop implications for the equilibrium convenience yield. This is unfortunate, since most of the time series models described later assume some structure for the convenience yield, and contrary to the literature on bond yields,[57] one cannot generally tell whether such structural assumptions are commensurate with rigorous economic equilibrium.

3.1.4. Time Series Models

Time series models of commodity futures attempt to “guess” at an appropriate model that fits the stylized facts of the particular commodity. Usually, the model is presented in terms of a probabilistic process in the price and other variables, that is used to calculate the risk-adjusted probabilities in commodity

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[56] A time-dependent variable undergoes a geometric brownian motion when, over short periods of time, the change in its logarithm is normally distributed with mean and variance proportional to the size of the time interval.

[57] Cox, Ingersoll and Ross (1985a,b) derive restrictions on admissible short-term interest rate processes under their general equilibrium model. These restrictions have more or less defined the types of models now used in practice.

market state prices. Occasionally, the process used to determine the true probabilities is also presented.[58]

The relationship between true and risk-adjusted processes may be discussed at many different levels. Bradley (1998), Cox, Ingersoll and Ross (1985a) and Duffie (1996), and the references contained therein, provide such a discussion from the simplest to the most general and "correct".

A key issue in model choice is usually tractability. The most popular models used in MAP are linear in the explanatory variables. The first advantage of such models is that the risk-adjusted probabilities for the spot price are lognormally distributed, leading to analytic formulae for futures and forward prices (and prices of simple European options).

The second advantage of linear models is econometric. Since a universal spot price is not generally observable for energy commodities,[59] and calculated futures prices depend on the spot, parameter estimation involves an unobserved state variable. The method of choice in such circumstances is the Kalman filter (Schwartz 1997, Hamilton 1994), which consistently estimates the model parameters along with the unobserved variable. More importantly, an estimation error is given on the unobserved spot price that, in turn, allows the practitioner to place error bounds on a MAP valuation. Another important advantage of the Kalman filter is that it allows researchers to estimate the risk premium associated with futures and forward contracts.[60]

3.1.4.1. One-Factor Models

Until about 1990, with few exceptions,[61] applications of MAP to project valuation relied exclusively on the Brennan-Schwartz (1985) price model. This is a single-factor geometric brownian motion model for the risk-adjusted price evolution, with a constant convenience yield, δ, and constant risk-free return, r. It originates from the Black-Scholes-Merton formulation of equity prices for a stock that gives a constant dividend yield.

In this model, the futures price for a contract maturing after a time period, τ, is $F(\tau) = Se^{(r-\delta)\tau}$, where S is the current spot price. Thus the term structure is in

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[58] This is useful if there is information about the true distributions that can be used to parameterise or check the model.

[59] Spot prices vary with the geographic location and spot markets are often illiquid.

[60] The risk premia estimated this way often have large errors, consistent with the inconclusive empirical findings on the risk/return theories discussed in Section 3.1.31.

[61] Laughton (1988) is one exception.

perpetual weak backwardation, if δ is positive, and in strong backwardation, if $\delta > r$. It should be clear that this model captures only the effects of backwardation and virtually no other property of energy commodity futures listed earlier.

Moreover, a simple extrapolation of the short-term uncertainty to longer terms using this model greatly exaggerates the amount of long-term uncertainty if there are any long-term equilibrating forces in the commodity market (Laughton and Jacoby 1993, Salahor 1998, Bradley 1998, Smith and McCardle 1999). These authors show how this can cause significant errors in asset valuation and the determination of optimal asset management policies.

To take long-term equilibrating forces into account, they use the Ornstein-Uhlenbeck reverting process with a target price scenario. There is a force, proportional to the logarithm of the ratio of the price to its target, that tends to drive the price toward the target. If the spot price is above the scenario price, then the forward curve is in backwardation; if it is below, there is contango. The reversion simulates the Kaldor-Working effect[62] and the stationarity[63] of the model guarantees a Samuelson effect.

In this model, American options are not exercised if the spot price is much below the target, even if the variable costs of production are zero. Laughton (1998d) notes that the problem is even worse, in that, with reasonable model parameters, a stockpiled commodity would be withheld from the market at prices significantly above the target price. Another problem with this model is that many commodity prices are not stationary (crude oil, in particular).

Other one-factor models are discussed in Brennan (1991).

3.1.4.2. Multi-Factor Models

A multi-factor model for energy commodities is presented in Laughton (1988). Oil price expectations are modelled to evolve according to a two-dimensional geometric brownian motion with a long-term factor and a short-term factor. Forward prices are calculated by discounting the price expectations according to the uncertainties in each factor using constant prices of risk. The initial price expectations, the volatilities in the two factors and the prices of risk are set to reproduce term structures, supplied by expert opinion, for forward

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[62] High inventories imply low prices, which produces a contango or a small convenience yield. Shortages imply high prices and backwardation or a large convenience yield.

[63] A price series is stationary if its moments (e.g., mean, variance), in the limit of large time, are independent of time

prices and price fractiles conditioned by different price scenarios. His model allows for reversion or overshoot in the short-term factor.

Gibson and Schwartz (1990) present a two-factor model where the risk-adjusted spot price follows a geometric brownian motion and the convenience yield follows a correlated reverting process. Schwartz (1997) demonstrates that this model is very effective in fitting the forward term structure of crude oil.

As noted in Section 2.4, this model is equivalent to one in which the commodity price experiences two types of risk-adjusted shocks: persistent movements that cause the entire futures curve to shift and a decaying component that only affects the nearby contracts (Schwartz and Smith 2000). In its various guises, it is deemed by practitioners to be the most versatile model currently available for valuation of oil assets (Baker, Mayfield and Parsons 1998).

A similar, more theoretically motivated, model is suggested by Routledge, Seppi and Spatt (2000) as a particular example of their equilibrium model.

The equilibrium model of Carlson, Khokher and Titman (2000) is very similar to the Schwartz and Smith (2000) model, but the tails of its price distributions are truncated when compared to those of Schwartz and Smith (2000).

Models that increase the number of factors to achieve a better fit are discussed in Schwartz (1997), Schwartz and Smith (2000), Cortazar, Schwartz and Riera (2000), and Miltersen and Schwartz (1998). The first includes interest rate effects, the second and thrid a stochastically varying long-term growth rate, and the last a Heath-Jarrow-Morton (1992) approach to modeling futures and forward term structure.[64]

Hilliard and Reis (1998) add jump diffusions[65] in their model. A difficulty with such models is the general failure of the replication methods for determining price relationships. Unless the jump amplitude is non-random, it is impossible to match the risk of one asset by holding some proportion of another in a portfolio (Merton 1976). To obtain state prices, Hilliard and Reis assume that the excess risk in the option due to jumps is not priced. Because jumps are an important feature of certain energy commodities (especially electricity), a better approach would deduce the risk adjustment from a general equilibrium model, as in Routledge, Seppi and Spatt (2000).

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[64] Instead of conjecturing a spot process with an associated risk premium and deriving the futures term structure, Heath, Jarrow and Morton (1992) directly use the term structure curve itself as the scenario state variable.

[65] These correspond to discontinuities in the price series that are distributed according to a Poisson distribution with (possibly) random jump amplitudes.

None of the models discussed thus far include uncertain price volatility. Although Duffie and Gray (1995) discuss such models with regards to energy commodity spot prices, there is not much in the literature at this point on how heteroskedasticity affects the futures curve or what ought to be the correlation between convenience yield and volatility. Some noteworthy exceptions are Litzenberger and Rabinowitz (1995) who relate backwardation to a put option on the commodity spot; the convenience yield in their model is positively correlated with the price volatility. In Routledge, Seppi and Spatt (2000), heteroskedasticity in the demand function automatically enters in the calculation of the forward curves and thus the convenience yield.

3.1.5. Special Issues with Electricity

Electricity is a special commodity in that it cannot be stored and can be derived from some other priced energy source. Kaminski (1997) discusses several features of electricity prices that result from the economics of this commodity.

Sudden shortages cannot be quickly smoothed because of the lack of large-scale storage. This leads to price spikes with quick reversion to a floor level. Volatility also tends to be higher (lower) when prices are higher (lower). Finally, the derived nature of the commodity leads to cross-correlations with its fuel prices. Routledge, Seppi and Spatt (1999) derive an equilibrium model featuring the above properties that can be used for valuation and pricing of derivatives.

3.1.6. Concluding Remarks

The last decade has seen much progress both theoretically as well as technically in modeling commodity futures markets. The models developed by researchers are more sophisticated and account for more of the stylized facts. The theories are better understood and are beginning to touch base with the needs of practitioners. There is still, however, a great deal of progress to be made. On the theoretical and econometric side, it is important to explore whether one type of theory is enough or whether a hybrid is necessary in capturing the essentials. Regarding the price models, better contact with equilibrium theory is needed. One potential danger with partial equilibrium approaches is that they may actually lead the market itself to systematic bias. Since most of the trading in the illiquid long term futures is conducted by firms with large multi-year projects, it could be that prices for such illiquid contracts are not determined by competitive markets, but rather by formulae derived from “popular” price models.[66] In such a case, searching for the best fitting model to futures prices becomes a self-fulfilling venture.

3.2. Other Priced Risks

There has been little serious work done on how to determine the state prices of other types of priced risks, such as those associated with commodities that are not traded in forward/future markets,[67] or different types of price indices that do not involve a single commodity. Systematic investigations are needed into how to use information in prices of any relevant corporate securities and, more importantly, expert opinion to specify and to parameterise models that include such variables.

3.3. Unpriced Risks

Local project-level uncertainties influence state prices only through the true probabilities involved, without any risk-adjustments. It is not clear at the present time what special problems will arise from the incorporation into MAP models of insights from fields like geostatistics, reservoir modelling, project planning, and technological forecasting. Work published to date has been relatively simplistic and usually for demonstration purposes only. The literature has not yet dealt with the issue of the constraints on modelling and the reliability of results that arise from the methods, and types of data, available to determine state probabilities for states associated with these local uncertainties.

4. Doing the Calculations

4.1. Linear Cash-flow Models

The calculations are trivial in the case of linear cash-flows once the forward prices are determined. If the cash-flows are modelled to occur at periodic intervals as in the typical DCF evaluation, the valuation parallels a single scenario DCF calculation exactly (Salahor 1998), and can be done easily with an electronic spreadsheet. There is only one difference. In the MAP valuation, the cash-flows in the forward price scenario (i.e., the scenario where each uncertain input into the cash-flow model is set to its forward price) are discounted for time. In the DCF valuation, the cash-flows in the forecast scenario are discounted for both risk and time.

[66] Trading in such contracts is typically done by large firms that often have a division specializing in pricing derivative assets.

[67] An exception to this is Routledge, Seppi and Spatt (2000).

4.2. Other Situations Without Real Options

In these situations, there is a single sum over states of the product of the state cash-flow and state price. The determination of the state cash-flow, for the states along a given scenario, is the same as the simulation of the cash-flow in that scenario. If the cash-flows are modelled to occur at periodic intervals as in the typical DCF evaluation, this simulation parallels that in a DCF analysis exactly. The state price determination is the equivalent of the computation of discount factors in the DCF analysis and the probabilities of the scenarios (Bradley 1998). Therefore a state pricing analysis is no more computationally intensive than the typical Expected Net Present Value (ENPV) analysis, and can be managed in a similar way.

If cash-flows occur at discrete times, then the set of states, over which the sum must be done, is finite-dimensional. Therefore a finite-dimensional integration can be used to do this sum, if a formula for the one-period state prices can be determined.[68]

There are many ways to perform this integration. One efficient method of high-dimensional integration is to sample the space randomly and take the sample mean. There are different sampling techniques. Because of its simplicity, the sampling technique most commonly reported in the literature is Monte Carlo sampling (e.g., Bradley 1998, Jacoby and Laughton 1992).

4.3. Real Options

ROV adds the search for optimal management policies to the computational mix. The policy set is usually very large, and efficient search techniques are required.

In the literature thus far, dynamic programming methods, which we describe in Section 1.4.3, have been used almost exclusively to implement the search for optimal policies. We focus most of our discussion on the two most common alternatives discussed in the literature for setting up an ROV dynamic programme, before making some brief comments about a potential alternative.

Some readers may be daunted by the numerical analysis that is involved in some of these calculations. However, if the use of MAP techniques continues

[68] One-period state prices can be calculated for linear time-series models, which, as we note in Section 3.1.4, give lognormal risk-adjusted probabilities. We know of no "real-world" applications that have gone outside this paradigm as yet.

to grow, commercial software will be made available to manage the technical aspects of the analysis, and possibly to assist in the modelling aspects as well. The "number crunching" can be managed in much the same way as goal seeking, optimisation, linear programming, statistical analysis, and random sampling are handled either directly as features within spreadsheet programmes or through add-ins to those programmes. The use of commercial software to support modelling in other domains is well advanced. This can occur for ROV as well.

4.3.1. Standard Dynamic Programming: General Comments

At each state in the scenario tree, the computational aspects of a dynamic programme are:

1) the determination of the value of the asset for each action available at that state, where the value is the sum of:
    a) the cash-flow resulting from that action in that state; and
    b) the value stemming from each follow-on state, if any, for that action; and
2) the determination of the most valuable action(s).

The scenario trees used thus far in most ROV applications are analogues of the trees used by Black and Scholes (1973) and Merton (1973) in their original MAP applications. They represent, in whole or in part, the resolution in continuous time of the uncertainty in a finite set of continuous time-series variables involving priced risks. The risk-adjusted movements in these variables are self-contained and Markov (i.e., depend at any given time only on the variables at that time). The uncertainty in the movements is compact enough to allow for replication. As a result, at each instant in continuous time, there is a single- (or multi-) dimensional continuum of recombined states, labelled by the variable(s) involved. We call these variables the "underlying" variables, and their states the "underlying" states, of the analysis.

These underlying states may be split up further into states for any needed auxiliary variables (e.g., the remaining reserves in Brennan and Schwartz 1985), and any unpriced risks. We call these states the "unpriced/ auxiliary" states.

The policy sets used in most applications are labelled by boundaries between regions in the state space where different actions are taken. The search is made for these boundaries.

For most applications reported thus far, the sum over states is done by numerical integration. Accuracy, stability, efficiency and generality are important considerations in choosing a computational method for this purpose.[69] The methods most often used in reported applications are: [70]

1) summing over the states in a discrete recombining tree representation of the connections between states at adjacent times; and
2) solving (linked) partial differential equation(s) using a finite difference approximation.

Both of these types of methods currently restrict the dimension of the underlying states, for practical purposes, to at most three. We return to the issue of this restriction in Section 4.3.4.

Before discussing each of these two types of computation, we should say that, generally speaking, we prefer the PDE approach, which can be used systematically in a wide variety of situations. The recombining tree approach is more ad hoc at this point. This might change if a more comprehensive theory for it is developed. Unfortunately, to our knowledge, this theory does not yet exist.

4.3.2. Discrete Recombining Tree Methods

Cox, Ross and Rubinstein (CRR) (1979) is the seminal paper on the recombining tree method of performing state pricing calculations. CRR (1979) use it to calculate the value of financial assets. The method is based on a discrete approximation to the underlying continuous scenario tree.

We shall focus first on situations with one underlying variable, where the risk-adjusted process for movement of this variable through time is a single-factor geometric brownian motion. This is the dynamics of the underlying variable in the original work of Black and Scholes (1973) and Merton (1973), in the early MAP work mentioned in Section 2.1, and in the Brennan-Schwartz (1985) price model.

The time periods in the discrete approximation are small. The continuous uncertainty at any state is represented in each of these small time periods by two branches. As a result, the methods are called “binomial” methods, or “binomial tree” or “binomial lattice” methods. Each branch is designed so that the magnitude of the underlying variable in the state to which the branch leads is proportional to the magnitude of the underlying variable in the state

[69] There are always small errors in the calculations. If the computational method is stable these errors are controlled. If not, they can be magnified. The efficiency of the method is a measure of the computational effort for a given level of precision.

[70] Smith and McCardle (1999) are an exception. They use linear programming methods.

from which it originates. If the constants of proportionality are time-independent, the branching can be designed to recombine. The same number of up and down movements result in the underlying variable having the same magnitude, and thus being in the same recombined state, independent of the order in which the movements occur. CRR (1979) show how to calculate the risk-adjusted probabilities on this approximate tree, so that in the limit of small time steps, they converge to the continuous-time risk-adjusted probabilities.

If the asset cash-flow in any state depends only on the contemporaneous underlying variable labelling the state and the action undertaken at that state, the dynamic programming calculations can then be performed on this discrete approximation of the continuous tree, using the CRR risk-adjusted probabilities. Trigeorgis (1996) describes the stability properties of this process and a logarithmic version of it. To be stable, both require limits on the size of the time step.

We should note that recombination of the approximate tree is a crucial feature of this type of computational method. With recombination, the number of states grows with the number of time steps in the situations we have discussed thus far. Without it, the number explodes exponentially, making computation with small time steps very difficult without some further approximation.

If we move to situations beyond this class of models, or if the cash-flow depends on the history of the scenarios in which they occur, the construction of the approximating tree becomes more complex.

Care must be taken to maintain the recombination of the tree, if the proportional expected movement of the underlying variable, or the proportional uncertainty in that movement, is time or state dependent (e.g., if there is reversion). Hull and White (1994a) present a method with three branches at each state that they apply to computations in situations where there is a one-factor risk-adjusted process for the underlying variable that exhibits reversion.

If there are multiple factors in the process, the binomial method must be expanded to deal with the multiple sources of uncertainty. The number of states grows more rapidly with the number of time steps. Boyle, Evnine and Gibbs (1989) present a method that is designed to work with multi-factor geometric brownian motions. The number of branches at each state in their approximate tree is 2^N where N is the dimension of the process. Hull and White (1994b) present a method with nine branches for a two-factor process with reversion. In each case, the number of states grows with number of time steps, T, as T^N.

One final comment. The trees used in this computational method do not generally converge to the continuous trees that they approximate. For example, the number of assets needed in the replication process for determining state prices should be the same in the discrete trees and the continuous tree, if the former are converging to the latter as trees (as opposed to merely providing a convergence of state pricing calculations). For the trees determined by a multi-factor process, the required number of replicating assets is one more than the number of factors (Duffie and Huang 1985). For the discrete trees, the required number of replicating assets is the number of branches emanating from each state (Duffie and Huang 1985). These numbers are two for both the binomial trees and the one-factor continuous trees that they approximate. In the other situations we have discussed, the number for the discrete trees is different from the number for the continuous tree to which they are intended to converge. Therefore, while the state pricing calculation may be converging properly, the trees, in some sense, are not.

4.3.3. Finite Difference Partial Differential Equation Methods

The other class of computation methods that has been used extensively is based on the solution of linked partial differential equations (PDEs).[71]

The relationship between the integral and the PDE representation of the value is discussed in several textbooks (e.g., Neftci 1996, Duffie 1996) and a variety of papers (e.g., Cox, Ingersoll and Ross 1985), with various levels of generality and mathematical precision.

We shall focus first again on situations where there is one underlying variable, and where the risk-adjusted process for its movement through time is a single-factor geometric brownian motion. Each PDE to be solved in this type of situation has the following basic form[72] at time t if the underlying variable is x:

$$V_t + 1/2\, \sigma^2 x^2 V_{xx} + (r - \delta)\, x\, V_x - r\, V = 0 \qquad (4)$$

[71] Finding the solution to a differential equation with boundary conditions is a form of integration. The fundamental theorem of calculus states that, under certain conditions, and with the appropriate boundary condition, the integration of the derivative of a function reproduces the function. Solving a PDE is akin to this process.

[72] If there are any continuously evolving auxiliary variables or continuously evolving variables involving unpriced risks, terms reflecting the effect of this evolution on the value are added to this equation. If cash-flow is continuous, the cash-flow is also added as a term. Brennan and Schwartz (1985) provide an example of a continuously evolving auxiliary variable and continuous cash-flow. The details are discussed later in this section.

where:

1) V is the value being determined, considered as a function of x and t;
2) V_t is the derivative of V with respect to t;
3) V_x is the derivative of V with respect to x;
4) V_{xx} is the second derivative of V with respect to x;
5) r is the risk-free interest rate;
6) δ is the "rate-of-return shortfall" in the expected movement of the underlying variable;[73] and
7) σ is the proportional uncertainty in x.

Unfortunately, there is no reference as yet that discusses, from the point of view of ROV applications, the various methods that may be used to solve such equations. However, there are several books about their use in the calculation of financial derivative asset values, where the issues involved, while not exactly the same, are similar. A good example of such a book is Wilmott, Howison and Dewynne (1995).

All the solution methods, upon which we shall report, use a rectangular lattice in t and x (or some transform of x) to set up finite difference approximations to the derivatives in the PDE. They all proceed by stepping backward through time, from a known terminal condition, in discrete time steps, using a simple finite difference version of the time derivative term in the PDE to do so.

There are several considerations about how this computational process is managed.

The first is the choice of representation for the underlying state variable, x. Some methods transform the equation so that the underlying variable is replaced by its logarithm. The coefficients of the derivatives with respect to the logarithm are constants in the transformed version of Eqn. (4).

Second is the choice of finite difference scheme used to approximate the derivatives with respect to the (possibly transformed) underlying variable. The usual choice is a centred difference for the first derivative and a three-point representation for the second derivative.

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[73] The "rate-of-return shortfall" is the difference between:

1) the equilibrium expected rate of return for a claim to a cash-flow determined by the magnitude of x in the near future; and
2) the expected rate of change in x itself.

It is the convenience yield if x is a commodity price, and the dividend yield if x is an financial asset price.

Third is whether these finite differences will be on the time slice from which the time step is being taken, the slice to which it is being taken, or some combination of the two.

The first of these alternatives gives what is known as an “explicit” method. At the lattice points used, the value is already calculated, and the determination of the finite differences may be done using these values. This makes the calculations for the value on any given time slice relatively fast. Unfortunately, the method is not guaranteed to be stable and this choice should be avoided if possible.

The other choices result in various types of “implicit” methods. These methods are stable, but they require the solution of a system of linear equations into which the PDE and its boundary conditions are transformed by finite differencing. Each of these equations is associated with a particular non-boundary lattice point on the new time slice. The unknowns in each of these equations are the asset value at this point and at the two points on either side. The relevant boundary condition is used to eliminate the value at the boundary as an unknown in the equations associated with the lattice points adjacent to the lattice boundaries.

This system of equations may be solved by a simple row reduction and back substitution, or iteratively from a trial solution where the value at the side lattice point(s) is determined from prior trials as required. Wilmott, Howison and Dewynne (1995) give a good discussion of these methods, using the representation based on the logarithm of the underlying variable. We have one reservation about this discussion to which we return later in this section.

The fourth and last consideration is whether the cash-flows and decisions are constrained to occur at discrete intervals or whether they can occur at any time. As we mention in Section 2.3, this can have important ramifications for the ease of computation.

If cash-flows and decisions are constrained to occur at discrete intervals, then the dynamic programme can be solved interval by interval, moving backward from one discrete time to the next.

Laughton (1998c) describes in detail how this is done[74] in situations where:

1) there is no change in variables with unpriced risks during the interval between the two discrete times;[75] and

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[74] This discussion occurs in his description of the development/production phase of an offshore oil-field.

2) any change in any auxiliary variables depends only on the unpriced/auxiliary state at the beginning of the interval and the action taken, and not on the underlying state at that time.[76]

We expand his discussion by relaxing these two conditions.

First, the value is calculated, for any possible unpriced/auxiliary state at the terminal time, as a function of the underlying variable. Then the following backward time step is performed for each discrete time interval, between the times when cash-flows and management decisions are modelled to occur, until the initial time of the analysis is reached.

Valuation at the beginning of each time interval involves solving a PDE for each possible unpriced/auxiliary state that may occur at the end of the interval. The solution "associated with this unpriced/auxiliary state" is found by using the finite difference approximation of the PDE to step back in time. The terminal condition is the value of the project in the unpriced/auxiliary state at the end of the interval. These PDE solutions provide the data for determining the follow-on value for each possible action in each state at the beginning of the interval.

In this process, there are fixed boundary conditions for the value at either end of the range of the underlying state variable. These boundary conditions must be chosen with care to be consistent with the dynamics of the value.

For example, we have the following considerations if the range of the underlying variable, x, is the non-negative real numbers (as it is if x is a price).

The boundary condition where x=0 should reflect the fact that, if x changes according to a geometric brownian motion, it remains zero, once it becomes zero. As a result, the value at x=0, at any time in the interval, should be the value at x=0 at the end of the interval, discounted risk-free for time.

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[75] In his model, the unpriced reserve uncertainty is resolved prior to development. If it were still being resolved, then there could be a change in a variable with unpriced risks during the time intervals involved.

[76] The auxiliary variables in this stage of his analysis are whether the field has been abandoned and, if it has not been abandoned, the time at which field development began. Each of these variables at the end of any interval is determined by what it is at the beginning, and possibly by management action then. They are not directly influenced by the underlying variable, the oil price, at the beginning of the interval.

This would not be so in a situation, for example, where there is a tax system with tax loss carryforwards. In this situation, the amount in the loss carryforward balance would be an auxiliary variable. Such a balance would depend, at the end of the interval, on both the balance and the oil price at the beginning.

A suitable boundary condition where x is large can usually be determined by information about the dependence of the value on x at large x, obtained independently of the solution of the PDE (possibly from asymptotic analysis of the underlying integral). In most situations, the second derivative of the value at large x is small compared to $1/x^2$. This boundary condition is implemented numerically, by choosing a large finite limit for the lattice of underlying states, and setting the finite-difference representation of V_{xx} to be zero there.

We now turn to the detailed use of these PDE solutions for the calculation of follow-on values in each state at the beginning of the time interval. It is useful at this point to introduce a little more notation. The time of the beginning of the time interval under consideration is called t, and the time at the end, t'. Each state at the time t is labelled (x,S), where x is the magnitude of the underlying variable in that state and S is the unpriced/auxiliary state involved.

If there is no resolution of any unpriced uncertainty during the interval [t,t'], then each state (x,S) at time t evolves, for a given action, A, into a unique unpriced/auxiliary state at time t', which we may call S'(x,S,A,t). The follow-on value at (x,S), given action A, is the PDE solution at (t,x) associated with the unpriced/auxiliary state S'(x,S,A,t).

If some unpriced uncertainty is resolved during the interval [t,t'], then, at each state (x,S) at time t, given an action A, there is a probability distribution prob'(x,S,A,t) for the possible unpriced/auxiliary states at time t'. In this situation, the follow-on value at the state (x,S) at time t, given action A, is the expectation with respect to the probability distribution, prob'(x,S,A,t), of the PDE solution at (t,x) associated with each possible unpriced/auxiliary state at time t'.

If we approximate a large, possibly infinite, set of the unpriced/auxiliary states at time t', by a smaller finite set (to make the computations feasible), then an interpolation among, or extrapolation from, the resulting set of PDE solutions may be needed to determine follow-on values at time t.

Once the follow-on value is calculated for each action in each state at the beginning of the given time interval, the cash-flow from that action in that state can be added to find the total value for that action in that state. Then the maximum value (and the actions that give it) can be found for each state. When this is completed, the process begins again for the next time interval unless the initial time of the analysis has been reached.

We now turn to situations where cash-flows and decisions can occur at any point in the continuum of time. This complicates things enormously. Brennan and Schwartz (1985) provide a good example of these complications.

First, if the policy to be found can be represented by boundaries between regions of states where different actions are taken, we must find these "free" boundaries continuously in time. This continuity imposes conditions on the boundaries so that, at any given time, they must be found consistently with each other (if there is more than one boundary) and with the valuation at that time. One form of these conditions is the "high-contact" condition (Merton 1973, Dixit 1993), which requires that the derivative of the value at the boundary be continuous.[77]

This restricts the type of algorithm that can be used to solve the PDEs involved. Wilmott, Howison and Dewynne (1995) give a good discussion of one such algorithm, the projected successive over-relaxation (SOR) method, which is an extension of one of the iterative finite difference methods for solving PDEs with fixed boundaries. There must also be consistency among the valuations in different regions, which further complicates the algorithms. For example, Brennan and Schwartz (1985) deal with the requirement that the valuation of the open mine must be consistent with the valuation of the closed mine (each provides a boundary condition for the other) through an iterative calculation of the pair.

Second, and more seriously, some auxiliary variables and variables with unpriced risks may evolve continuously in time, if cash-flows occur, and decisions are made, continuously. An example is the remaining ore reserves in the Brennan-Schwartz (1985) analysis. Because mining takes place continuously when the mine is open, the value of the open mine changes continuously with the continuous changes in the remaining reserves. This introduces new terms into the PDE for the value of the open mine, involving derivatives of the value with respect to the remaining reserves. In the Brennan-Schwartz (1985) analysis, the amount of the remaining reserves has no independent uncertainty, so the only term that is introduced is in the first derivative. This means that the PDE is not a "parabolic" PDE, which is relatively easy to solve numerically, but a "hyperbolic" PDE, which is much more difficult.[78] [79]

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[77] Merton (1973) implicitly uses the regularity of the boundaries in time to show that the high contact condition is required. If decisions occur, and thus the boundaries exist, only at discrete times, this condition is not required, contrary to the implication of Wilmott, Howison and Dewynne (1995). This is our reservation, mentioned above, about their discussion of these computations.

[78] For information about the classification of PDEs and the solution of hyperbolic PDEs, see, for example, Press et al. (1988).

[79] Brennan and Schwartz (1985) finesse this problem by using a time-independent formulation to eliminate time from their problem. The amount of remaining reserves then takes on the role of time in their analysis. It flows when the mine is open and is stalled when it is closed. The resulting open mine equation is parabolic and the closed mine equation is elliptic.

Some analyses use a mixture of discrete and continuous time analysis when this simplifies the problem. Laughton (1998c) is an example, where the cash-flows and decisions for an offshore oilfield occur annually once development is begun, while decisions about the timing of exploration, delineation and development occur in continuous time.

The computational issues are not very different for situations where the underlying variables follow a risk-adjusted one-factor process that is not a geometric brownian motion. We would like to make just two additional observations

1) If the process is reverting like that used by Laughton and Jacoby (1993) and Smith and McCardle (1999), care must be taken with the singularity in the expected change in the underlying variable that exists when it is close to zero. This is usually done by bounding the numerical analysis away from where the singularity occurs.

2) The usefulness of the logarithmic transform in the underlying variable is less when the coefficients of the resulting PDE are not constant.

If the underlying variables follow a multi-dimensional risk-adjusted process, the computations become much more complex for several reasons.

1) The number of underlying states to consider grows with the power of the dimension of the state space.

2) Stable finite-difference PDE methods for multi-dimensional state spaces are more complex. Two examples are alternating direction methods and hopscotch methods.[80]

   Alternating direction methods retain the same type of system of linear equations at each time step with two or three adjacent unknowns in each equation. This is done by making the finite differences explicit in all directions in the state space but one. The variable that is treated implicitly is changed with each time step, cycling through all of the variables, one per step.

   Hopscotch methods divide the underlying variable lattice into two alternating sets of points, in the equivalent of a chequerboard-like pattern, and alternate, over time steps, which set of points have their finite differences treated implicitly, with the others being treated explicitly. It turns out that no matrix inversion is required, and the method looks “explicit”, even though it is stable.

[80] For a fuller discussion of these methods, see, for example, Wilmott (1998).

We know of no systematic study of the efficiency of these methods for typical ROV problems.

3) The policies are no longer represented by single-point boundaries at each time, but boundaries that are curves (if the state space is two-dimensional), sheets (three) and hypersheets (more than three).

4.3.4 Limitations of Current Computational Methods and the Potential of Random Sampling Methods

There are serious limitations to the PDE approach that will become important as users demand:

1) increasing reality in the modelling of:
    a) uncertain inputs into their analyses; and
    b) the project structures that they use; and
2) more modelling flexibility to address complex decision issues.

This will require analyses with larger underlying state spaces, more unpriced/auxiliary states, and a broader range of future decision alternatives. At present, computational feasibility imposes a practical limitation on the size of the state spaces: a two- or at most three-dimensional underlying state space, and a few thousand or few hundred unpriced/auxiliary states (depending on the dimension of the underlying state space). This is a severe limit on modelling flexibility. While a shift to discrete-time cash-flows and decisions expands the range of analysis greatly beyond what would be available under the strict continuous-time approach, much more needs to be done.

One rather pedestrian approach that should help would involve the more systematic use of interpolation among, and extrapolation beyond, PDE solutions for different unpriced/auxiliary states. This should reduce the number of PDE solutions needed to cover a given situation properly.

A more ambitious approach is to combine random sampling methods, which can sample high-dimensional state spaces, with different search techniques in creative ways to find optimal policies for managing asserts with cash-flows that depend on a large number of underlying variables. The early literature is reviewed in Boyle, Broadie and Glasserman (1997). Two recent examples of the growing number of more recent papers are Longstaff and Schwartz (1999) and Garcia (2000). Both look at an option to receive a payoff with a known dependence on a large number of uncertain variables where the timing is optional.

Longstaff and Schwartz (1999) work backward in time discretely along a random sample of scenarios to find when the option should be exercised in each scenario. The value of waiting at each time in each scenario is determined by a regressing, across the sample of the scenarios, the future value of waiting against a finite set of basis functions for an infinite-dimensional space of functions on the underlying state space.

A different approach is taken by Garcia (2000), who finds the value by bracketing it between high and low estimators. Rather than parameterising the "waiting value", he parameterises a finite-dimensional set of functional forms for the boundary of the set where the option is exercised. He then maximises a finite-sample state-pricing valuation for the option over the finite-dimensional set of parameters for this boundary. This is his high estimator. His low estimator uses a different independently determined finite sample to estimate the state pricing value with this same boundary.

There is a lot of work that must be done to make these sort of methods useful for ROV applications, but they show some promise for expanding our computational capabilities toward the analysis of more realistic models of real assets.

5. Some Applications

We now turn to a brief description of some publicly presented applications of MAP in the energy industry. Most of these applications have been in the upstream petroleum industry, from the viewpoint of an organisation exploring for, developing or producing petroleum.[81] We also report on some applications of relevance to electricity generation and distribution.

5.1. Upstream Petroleum Applications

5.1.1. Non-ROV Applications

As noted in Section 1.5, the simplest applications of MAP do not have options to be analysed within the context of the valuation. This is the case for all of the applications discussed in this section (5.1.1). All of the analyses here use the simple valuation method discussed in Sections 1.4.1 and 4.1, if the cash-flow model is linear in the oil and gas prices. If it is not, they use Monte Carlo sampling to implement a state pricing valuation, as discussed in Sections 1.4.2 and 4.2.

[81] The issues faced by a resource owner, particularly a public resource owner, or a petroleum transport organisation are different from the typical developer.

Bradley (1998), Jacoby and Laughton (1992), Laughton (1988) and Salahor (1998) show how different production and cost patterns can influence project-level risk in situations where the revenues are more risky than the costs. Under these circumstances, there is operating leverage and project net cash-flow is more risky than either the revenues or the costs.

Salahor (1998) shows how Standard DCF analyses, performed with a discount rate high enough to be appropriate to discount for project risks, can seriously distort decisions that involve tradeoffs only in the time pattern of costs. Because the costs have less risk than the project cash-flows, a project discount rate is too high for these purposes. In Salahor's example, Standard DCF methods undervalue the use of up-front capital to decrease later operating costs.

Bradley (1998), Jacoby and Laughton (1992) and Laughton (1988) compare development projects of different sizes. They show how, if there are economies of scale during production, the use of a single discount rate can significantly bias the analysis toward investment in smaller, riskier projects.

Bradley (1998), Laughton (1988) and Salahor (1998) examine the interaction of project structure and price reversion. Salahor (1998) explicitly shows how the value of long-term projects is underestimated when compared to the value of short-term projects, either by ignoring reversion in a MAP analysis or by performing a Standard DCF analysis with a single constant discount rate.

Bradley (1998), Jacoby and Laughton (1992), Laughton (1988), Lund (1992), Majd and Myers (1986) and Salahor (1998) all show how MAP can be used to examine the effect of fiscal terms on project value and risk. They all presume there is no political risk in the fiscal regime.

Salahor (1998) shows how an ad valorem royalty, when deductible for corporate income tax purposes, increases the risk of the corporate income tax. He also shows how the depreciation (rather than the expensing) of capital costs for tax purposes can have the same effect. Furthermore, he shows how the less risky royalty induces risk in the net cash-flow to the developer and how the more risky income tax absorbs it.

Bradley (1998), Laughton (1988) and Majd and Myers (1986) show how the income tax behaves at the project level if the corporation has no other income.[82] Laughton (1988) and Majd and Myers (1986) compare this to the situation with large other income, and Laughton (1988) also extends the analysis to the presence of risky other income. They make the point that the

[82] Bradley (1998) actually analyses an accounting profits royalty (APR), which is equivalent to a standard corporate income tax in the situations he examines. Laughton (1988) performs his analysis in the context of the old UK tax systems, in which there is a Petroleum Revenue Tax (PRT) and, in some cases, an ad valorem royalty

tax system is asymmetric in that profits are treated differently (tax payments are required immediately) from losses (tax refunds are not given immediately, but may be given later on a carryforward basis). Bradley (1998) shows how, for marginal projects, if the pretax valuation is kept constant, the value of the tax claim increases with the level of underlying uncertainty. With more uncertainty, there is more upside potential for the government to tax, while the downside does not result in corresponding tax decreases. Laughton (1988) shows how, in some circumstances, the marginal value of a project to a corporation with risky other income is higher than it would be to corporation with large other income against which early losses can be immediately deducted.

Jacoby and Laughton (1992) and Laughton (1988) do a similar analysis of the Petroleum Revenue Tax (PRT) in old UK tax system, with its ring fence, uplift and oil allowance. This is also an asymmetric tax, primarily because of the ring-fencing, but there are nonlinearities in the oil allowance as well. Both analyses show how the PRT absorbs risk away from the developer (and also from the income tax stream), especially for smaller projects if there are economies of scale. They also show how the oil allowance subsidises smaller fields and how the uplift provisions subsidise smaller fields if there are economies of scale in development costs.

Lund (1992) examines the effect of two versions of the Norwegian tax system on the upfront production capacity choice in the presence of a particular model of economies of scale. With this particular model, the tax system tends to decrease the optimal production capacity. He compares the effects of the two tax systems, and finds that the newer system causes less distortion.

Cavoulacos (1987) and Cavoulacos, Blitzer and Laughton (1988, 1992) give an example of how MAP might be used to examine the effects of political risk on project value. They look at the case of a small producing country that might change its regime for a particular project in response to windfalls to the developer from high oil prices or larger than expected reserves. They assume that there is a faction in the political system that favours expropriation. This faction gathers and loses strength as the ex post value to the developer waxes and wanes, increasing and decreasing the probability that a given contractual parameter will be changed unilaterally to reduce the ex post value to the developer. They compare the contractual stability and the willingness of outside investors to develop the field, if the fiscal regime is based on an ad valorem royalty, a production sharing agreement, a service contract or a resource rent tax. They briefly discuss the tradeoff in fiscal system design between stability and appropriate risk sharing.

5.1.2 ROV Applications

One of the first ROV applications in the upstream petroleum industry was the analysis of US offshore leases by Paddock, Siegel and Smith (1988).[83] As we note in Section 2.1, they use the value of the developed field as an underlying variable. They argue that the volatility in the value of the developed fields is the same as that for crude oil (i.e., that there is little, if any, operating or fiscal leverage in oil production). They calculate the rate-of-return shortfall for the developed reserves by assuming once again that there is no leverage or other differences in the benefits and costs of holding oil above or below ground. There is no reversion in their valuation model. They also presume that development begins immediately upon the end of exploration and that production begins after a predetermined lag, once development is begun. The exploration/development decision may be made at any time during the lease, and the owner may walk away from the lease. Their empirical results are mentioned in Section 6.

Pickles and Smith (1993) continue in this vein by allowing an option in the timing of development rather than exploration. They use a binomial lattice method for performing their valuations.

Bjerksund and Ekern (1990) use the Brennan-Schwartz (1985) price model to build simple models of:

1) fixed-time options to develop a field (for which they use the Black-Scholes formula); and
2) indefinite leases (for which they use the Merton (1993) formula for an indefinite call on a dividend-paying stock).

They structure their analysis so that they can use "closed-form" solutions as much as possible. While this constraint permits a qualitative discussion of certain features of the leases they consider, it limits their ability to consider a variety of project details that would be important in an actual evaluation.

Mackie-Mason (1990) also examines development leases, focusing on some effects of the percentage depletion allowance (PDA) in the US corporate tax system. He uses the Brennan-Schwartz (1985) price model in his analysis, and presumes there is no cost or production uncertainty. The PDA is subject to an income cap, which makes the tax cash-flow nonlinear in the oil price, even if there is large other income to absorb tax losses. He shows how an increase in the tax rate and a decrease in the depletion allowance rate can each decrease the boundary between the (higher) price region where immediate investment is more valuable and the (lower) price region where waiting is preferable. Each of these changes in the tax system decreases the

[83] A summary of this work is presented in Siegel, Smith and Paddock (1987).

value of investing in the project now. Despite this, each can encourage investment if the value of waiting is decreased even more.

Laughton and Jacoby (1991a,b)[84] show how to value development leases for complex projects where the cash-flows at any time after the "development time" (the time when the decision to develop is made) are influenced by the whole scenario after that time. They use, as examples, UK North Sea development leases, valued under a Brennan-Schwartz (1985) price model with no cost or production uncertainty. At any potential development state, they determine the "development value" (the payoff that would result from making the decision to develop) by interpolating among the development values calculated on a grid of such states. They use Monte Carlo methods to perform the high-dimensional sum over states that is required to determine the development value at each state on this grid. Then they use finite difference PDE methods to do a dynamic programming search for the lease value and the optimal development policy that would arise from the calculated pattern of development values. Finally, they use Monte Carlo methods once again to check the dynamic programming results, by determining the lease value for each of a grid of policies around the best policy found by dynamic programming.

The option to develop a satellite field is considered by Ekern (1988) in a highly stylised two-period model. He outlines an interesting choice between:

1) adding production and transport capacity to bring the satellite into production while the main field is still at peak production; and
2) waiting until the production in the main field has declined enough to free up capacity for use by the satellite.

Smith and McCardle (1999) also mention the option to use production and transport capacity for satellite fields in their analysis, but do not give explicit details about it. A more detailed publicly reported analysis of this type of problem would be interesting.

Laughton (1994) and Laine (1997) both consider the option to expand production by a fixed amount at any one time after the field has begun production. Both presume a Brennan-Schwartz (1985) price model and risk-free costs and production. To perform the valuation, Laughton (1994) uses finite-difference PDE methods, with annual cash-flows and decisions, while Laine (1997) uses binomial lattice methods in the logarithm of the price. Both allow the choice about abandonment to be made at in the future, and imbed their analysis into a development lease.

[84] Laughton and Jacoby (1991b) is an expanded version of Laughton and Jacoby (1991a). The analysis in Laughton and Jacoby (1991b) is done in real terms, which can simplify the calculations and the presentation enormously, when compared to the nominal presentation in Laughton and Jacoby (1991a).

Laughton (1994) presents the option values both for the five-year development lease and the "now-or-never" development choice. He also shows the critical prices for abandonment (contingent on the timing of expansion), for expansion and for development (with and without the expansion and abandonment options). As might be expected, he finds that the incremental value of the expansion option is greater at high prices and the abandonment option at low.

Laine (1997) looks at two fields and presents valuation results that have a similar qualitative structure, although it appears that the development lease that he considers does not allow the developer to walk away at the end of the lease.

Oguztoreli (1996) uses a calibrated 3-D black oil reservoir simulator to determine production profiles that will result from waterflooding, at different times in the future, an oil-field that is currently in primary production. He then uses this production data to perform an ROV analysis of the opportunity to initiate this waterflood at some point during a given period of time. He uses a Brennan-Schwartz (1985) model for the oil price, and treats cost and production profiles as known with certainty under any given management policy.

His work follows Hanson and Laughton (1994), who examine situations where the waterflood could fail. They broaden the policy set to include the possibility of a pilot waterflood. The pilot would have a lower average cost than the full waterflood, because it would not require new facilities. However, it would delay the full waterflood by a year. This delay can make the information gathering process costly at high prices (Laughton, Frimpong and Whiting 1993). In fact, the behaviour at high price is a tradeoff between the risk of lost production, if the waterflood fails, and the risk of delayed production, if it succeeds. There can be complex behaviour at low prices, and the incremental option values give interesting results.

Chorn and Carr (1997) examine exploration and production capacity decisions for an offshore gas prospect. Production must begin or the project be abandoned at a given fixed time in the future. The initial capacity decision may be made now without exploration, or deferred for a given period of time with current exploration optional. The subsequent capacity decision is made at another fixed time. Abandonment is possible, and only possible, at each time a capacity decision is made. They use a Brennan-Schwartz (1985) price model for the gas price and presume the costs are determined by the pattern of capacity and exploration decisions and the production rate. They presume that the expected reserves follows a controlled brownian motion, with a restriction that the expected reserves must be non-negative. The uncertainty in the change in the expected reserves is dependent on the level of production and whether exploration is taking place. There are lead times between the capacity decisions and when the capacity comes on line. The dynamic programming computations are done using a bivariate binomial lattice for the gas price and the expected reserves. Results are presented without full information about the inputs. They say that the incremental value of the exploration and capacity options can be as much as 28% of base value. They show how this value depends on price, the price volatility and the reduction in reserve uncertainty from exploration.

Smit (1997) looks at the exploration and development of three oil concessions on the Dutch continental shelf. They feature different amounts of reserve uncertainty. He uses three Brennan-Schwartz (1985) oil price models with different levels of volatility. Each concession has a finite life and is awarded on the basis of exploitation proposals. Test drilling to resolve dry-hole uncertainty occurs immediately or the project is abandoned. If the hole is found to be wet, appraisal drilling occurs at a fixed time in the future to determine reserve size, or the project is abandoned. If the appraisal is done, production and cost profiles are then known. It is not clear from his presentation whether the timing of development is optional. Abandonment can occur at any time during production and requires the outlay of a decommissioning cost. It is not clear whether a partially developed field can be abandoned. He includes taxes in the cash-flows for the development/production phase. Cash-flow during this phase is modelled to depend only on the abandonment status, the time since development was initiated, the current oil price and the reserve size. He uses a binomial lattice approach to perform the dynamic programming needed in the production phase. It is not clear what he does to compute the dynamic programme during the development phase and the two exploration phases. He states that, to find the appraisal value, he sums the value over reserve states, which are weighted by their true probability because the reserve risk is unpriced. He does not state how he performs the sum of these values over the oil price states in these phases. We presume it is by continuing to work through a binomial lattice. He observes that higher price volatility increases value in the face of the options presented, and that greater reserve uncertainty can increase the value of exploration activities.

Smith and McCardle (1999) also examine the life cycle of an off-shore oilfield development after substantial exploration has determined that there is an exploitable reserve. They use a reverting price model, and allow for unpriced cost and production profile uncertainty. They consider development timing and capacity decisions, with follow-on decisions to explore for and tie in satellite fields. They briefly describe the use of linear programming methods for doing the dynamic programming calculations. They compare the results obtained from an ROV evaluation and an equivalent DTA where the cash-flows are discounted using DCF methods with a single discount rate.

Laughton (1994, 1998c) analyses a petroleum project life cycle through exploration, delineation, development and production, with abandonment always possible. Laughton (1994) uses one Brennan-Schwartz (1985) price model, and Laughton (1998c) uses two. The two models in Laughton (1988c) have the same forward prices but different volatilities. There are two models of reserve uncertainty. Both have a possibility of a dry hole, which is resolved by exploration, and the same expected amount of reserves conditional on a wet hole. In one model, there is no wet-hole uncertainty, and in the other there is a discrete wet-hole uncertainty. In Laughton (1994), this wet-hole uncertainty is resolved in stages by exploration, delineation and development. In Laughton (1998c), it is resolved by delineation alone. In Laughton (1994), there is a production capacity choice to be made at development time. In Laughton (1998c), there is not. The computations are done using the same technique in both sets of analyses. We describe here only the details of Laughton (1998c).

First, he shows how the time of abandonment is determined in a MAP framework, if that time must be specified at the development time. He then shows how value is increased in each state by the existence of the option to decide year by year whether to abandon. He also shows the critical price boundary below which the abandonment option is exercised for each relevant reserve size. With the term structure of forward prices left unchanged, he shows how the option value increases, and the price boundary decreases, with increasing price volatility. Moreover, he shows how the boundary declines during the development phase as development costs are sunk and then increases again during production, as remaining reserves decrease, and the concurrent average cost of production increases. Finally, he finds there can be a large incremental value of being able to decide on abandonment each year as opposed to setting the abandonment time prior to development. He uses linear cash-flow methods to analyse the value of the field with a predetermined abandonment time. He uses finite-difference PDE methods, with annual cash-flows and decisions, for the ROV analysis of the abandonment option.

Once the development value (i.e., the value of the project at the time development is begun) is determined for each reserve state and oil price, he analyses, using finite difference PDE methods, the value of an infinitely long development lease, for each possible reserve state, as a function of concurrent oil price. In doing so, he determines the optimal development price boundary.

The result of delineation is the determination of the reserve size, if not already known, and the creation of the development lease. Therefore, the payoff from delineation, for any given oil price, is the sum over possible reserve states, of the product of the state price for that reserve state and the value of the development lease for that reserve state and oil price. Because the uncertainty in the reserve state, if any, is an unpriced risk, and because there is no time delay between the completion of delineation and the beginning of the development lease, each reserve state price is simply the probability that that reserve state will be realised at the end of delineation.

Once the payoff form delineation is determined for each oil price, he analyses, using finite difference PDE methods as required:

1) the payoff from the choice to begin delineation (which takes time and requires an upfront investment) as a function of the concurrent oil price; and
2) the value of an infinitely long delineation lease as a function of concurrent oil price, and the optimal delineation price boundary.

He then repeats the process used for determining the value and control of the delineation lease to determine the value and control of the exploration lease. The difference is that the result of exploration is the determination of whether is a dry hole or not, and the creation of a delineation lease (which has zero value, if the dry hole state is realised after exploration). Therefore, to determine the payoff from exploration, he uses the delineation lease values instead of the development lease values used in the determination of the payoff from delineation.

The results demonstrate that each of the leases are more valuable with increased price uncertainty. The exploration and delineation leases are also more valuable with increased geological uncertainty, if actions can be taken to respond to the resolution of that uncertainty. In this situation, the actions taken to respond are timing actions. The results also demonstrate that greater exogenous uncertainty (e.g., oil price uncertainty) decreases the number of states where action is taken (i.e., in this case, increases the optimal price boundary for action), by increasing the probability of failure once committed and the probability of higher payoffs from waiting. However, greater endogenous uncertainty increases the number of states where action is taken to resolve that uncertainty (i.e., in this case, decreases the optimal price boundary for action), by increasing the value of acting to do so.

5.2. Electric Power Generation and Distribution

One of the first applications of MAP in the electric power industry was Martzoukos and Teplitz-Sembitsky (1992). They present a model to analyse the option of deferring irreversible investment in a transmission line hook-up to a regional power grid. The alternative is a sequence of investments in diesel generators. In the model, both power demand and transmission costs are uncertain and growing. Investment in the transmission line hinges on a comparison between the diesel cost and the smaller operating costs associated with the transmission line. Their model demonstrates that, when demand and transmission costs are uncertain, it is optimal to delay transmission line investment beyond the date that is optimal when such uncertainty is absent.

The ability to protect investments from adverse price movements has also been studied. Kulatilaka (1993) compares investment in a single-fuel industrial steam boiler to one that can burn two fuels. There are comparable decisions in fuel choice for electricity generation. The dual-fuel boiler incorporates protection from adverse price movements because it allows the operator to choose between the cheaper of two fuels. This ability is shown to justify the additional capital costs of the dual-fuel boiler in some situations and that it is particularly valuable when the operator is indifferent between fuels. One interesting feature of this application is the attempt to finesse the issue of risk-adjustments by presuming they are the same for the two fuel prices and thus cancel in the ratio for fuel prices used as an underlying variable

Hsu (1999) develops two models for natural gas-fired generating plant investments in which the completed power plant may be temporarily closed in response to operating losses. Both models use “spark-spread” options to determine the value of the completed power plant. The first model considers the investment in a new power plant where the investment decision can be made at any time during a deferral period. The problem is structured as a compound option and a purported “heuristic” solution for investment is provided. The second model considers the purchase of an existing power plant that may be refurbished in the future with more efficient technology. This problem has the same structure as the first model except that management may also exchange the inefficient plant for a more efficient plant at some point in the future. An “heuristic” solution is provided that purports to approximate the conditions at which the existing plant should be refurbished.

The 1990 Clean Air Act in the United States established a trading system whereby rights to emit SO_2 could be purchased and sold. The purpose of this trading system is to allow companies to comply with increasingly rigorous pollution standards more efficiently by allowing each company the opportunity to meet the standard by either installing better pollution controls or purchasing pollution rights. The presence of an active market in pollution rights sets the value of being able to expel a ton of sulphur over one year into the atmosphere and allows companies to gauge more easily the value of pollution control investments. Herbelot (1992) develops a model in which the power plant owner must determine how to meet pollution regulation when the prices for pollution rights and coal are uncertain. In this model, the pollution standards can be achieved by installing a scrubber and burning only high-sulphur coal or by switching between high-sulphur and low-sulphur coal fuel as market conditions dictate. Flexibility in the method of achieving the pollution standards is shown to be valuable when compared to a base case of not managing pollution emissions and purchasing the required pollution rights on the open market. Edleson and Reinhardt (1995) develop a comparable model that provides with similar results.

Utility regulation in some instances has been characterised by the determination:

1) of a rate base from which future profits are calculated;
2) of the maximum allowable rate-of-return that the rate base may earn; and
3) of provisions for cost recovery for abandoned plants.

These provisions influence power plant investment by modifying the return distribution for these investments. Rate base calculations and maximum allowable rate-of-return provisions limit the upside potential of the plant while cost recovery considerations affect the possible downside losses. The influence of regulatory structure on power plant investment analysis has become more important with the increase in uncertainty regarding the business environment in which power plants operate.

Teisberg (1993, 1994) develops a model of regulation where value of the completed project is uncertain. The value of the completed project increases at an expected rate of return less a regulatory term associated with expected changes in cost allowances. The regulatory term is modeled as a function of completed project value such that the project value is reduced when profits are high and return enhanced when project value is low. Project management has the option at pre-determined time points to continue construction of the project, temporarily halt construction, or abandon the project for its salvage value and cost recovery provisions. The model shows that under heavy regulation (i.e. when upside and downside returns are substantially reduced) construction flexibility has little effect on project value. As deregulation occurs, the options to delay construction or abandon the project are shown to increase in value. Teisberg also notes that the option to limit downside risk through investment deferral or abandonment limits the value of regulation designed to offset downside losses. In addition, high cost recovery provisions for partially completed plants are shown to provide an incentive to start power plant construction but, perversely, also to abandon a partially completed plant as uncertainty about the completed plant is resolved.

Kathan (1995) develops an alternative model of power industry regulation where management has the choice at pre-determined intervals either of continuing project construction or abandoning the project. The underlying source of uncertainty for this model is also the completed project value. Regulation provisions for loss recovery and profit restriction provisions are defined explicitly. Loss recovery provisions entail reimbursement of a pre-specified portion of the sunk construction costs if the project is abandoned early, or the project value is less than total construction costs upon plant completion. Profit restrictions are structured such that there is a pre-specified upper boundary on the completed project value (e.g., the final project value cannot be more than 120% of the initial expected value of the completed project). The effects of regulation are analysed from a welfare economics perspective in that the distribution of project benefits between producer and consumer are compared. A partial level of cost recovery is shown to be necessary to induce investment in the project. However, high levels of cost recovery are not considered welfare optimal. Kathan also notes that construction profiles that defer most of such costs to the end of the construction period increase overall welfare but decrease the producer surplus.

6. Empirical Work

There have been few attempts to test MAP models. As we note in Section 5.1.2, Siegel, Smith and Paddock (1987) and Paddock, Siegel and Smith (1988) provide a qualitative examination of the use of market prices for developed reserves to value offshore petroleum leases in the Gulf of Mexico. They compare the results of several lease auctions to the value that would have been calculated by their option model. Their comparison showed that the option model provided a better prediction of lease value than the conventional DCF methods used by the US Geological survey.

Moel and Tufano (1998) have examined the annual opening and closing decisions of developed North American gold mines. Their study confirms many of the predictions from real options models, such as investment decision deferral.[85] It also indicates that the factors influencing the probability that a mine is open are related to the factors incorporated into MAP models (e.g., level and volatility of the gold price, the interest rate, fixed costs, variable costs and remaining reserves). Finally, their study also shows that there are firm-specific managerial factors that also affect the decision to close a mine.

7. Conclusions

We have come a long way, and have a long way to go, in developing MAP technology for project evaluation in the energy industry. In looking at how much more can and should be done, it should not be forgotten that current

[85] The decision to defer a management decision even though the current course of action is unprofitable is often called “hysteresis” in the literature. In the case of mine operation, management may defer the decision to close the mine temporarily, even though the operation is incurring an operating loss. It does so to avoid the costs of reopening if prices go up.

gaps in DCF technology are as wide, if not wider, than those in MAP. We should demand more of the new method, to compensate for the costs of introducing it, but not so much more as to abandon a promising new development.

Future developments in MAP are needed on three key fronts:

1) better modelling of scenarios, policy sets, cash-flows and state prices;
2) more efficient and easily used computational methods; and
3) more and better training and operational tools for implementing MAP methods in real organisations.

The first point raises several issues.

We need a better understanding of commodity prices (including interest rates, inflation rates and exchange rates) and the pricing of their risks, moving beyond the phenomenological models we now have.

We need to understand how to split up costs into manageable pieces. We need to know how to relate the cost streams we do choose to model to output prices. For projects that transform one form of marketed energy into another, how should the spread be modelled? How should other costs be modelled? Should there be rent effects, where the costs depend on the output prices? Should there be quasi-rent terms where the costs depend on the change in output prices? Should costs be related to general cost indices? How should the effects of overall technological progress be modelled?

We need to understand how most efficiently and effectively to model project-level risks, whether they be those that affect output possibilities or those that directly influence costs.

We need to understand better the trade-offs involved in setting up the policy sets that we consider. When is a timing option important as opposed to forcing a decision to occur at a fixed time? What is the best way of modelling crucial design choices?

We need broader methods for using more sources of information, both historical data and expert opinion, in determining the parameters of state prices

On the second point, we need to fine-tune the methods we now have, by finding out how we can use more discretisation without intolerable loss of accuracy. At the same time, we must search for a next generation of tools for larger models.

On the third point, we need a set of commercial-grade modelling and computational tools, and a better, more detailed teaching curricula, both of which are geared to helping organisations make a gradual transition from the use of Standard DCF methods to the broader MAP valuation framework being proposed.

All of the these three developments will go hand in hand as better models and methods make the analytical tools more flexible and accurate, and thus more attractive. This will increase the demand for the development of better software and training tools, the development of which will make the analyses more attractive and encourage the development of better models and methods. All of this will be driven in part from the opportunities on the supply side given by more computer power, further development of data from derivatives markets, and the training that managers will receive at leading schools of management. It will be driven in part on the demand side by the increasing complexity and riskiness of the economic world, which will require better tools for aiding decisions in the face of those risks.

Within the context of these trends, different organisations will have different cost-benefit tradeoffs as they consider whether and, if so, when and how to introduce MAP methods into their system for making decision about asset configuration. Those organisations that operate in a variety of environments (fiscal, geological, technical) with a heterogeneous menu of complex projects will benefit more form MAP than those with a menu of projects that are all very similar to each other. Those organisations, that have experience in innovating with DTA and ENPV, have experience with making hedging decisions, have a culture of innovation, and produce and use commodities with well-developed financial markets, are likely to find the cost of introducing MAP analysis less than those that have only used internal rate of return measures of value, and produce and use commodities that are not traded actively.

To close, it took at least 80 years to sort out DCF and make its use widespread, beginning from its first forms and uses in the railway industry in the late 19th century, where it assisted with decisions about cost cutting investments. It is likely to take a shorter, but still significant, time to sort out MAP. We hope this review helps in that process.

Appendix: List of Signals That We Have Found Useful

We have found, for our own purposes, some characteristics of publications in this field that signal their usefulness for learning more about how to apply MAP in the energy industry. We list some of these characteristics here.

1) An item in the MAP literature is generally more useful if it is clear about the effects of the current and ultimate limitations of MAP methods on any MAP analysis that it reports.
2) If an item is in the ROV literature, it helps if it is clear about the relationships among ROV, MAP and DTA. In particular, an item is less helpful if it suggests that it is about ROV, and then reverts to reporting only on DTAs of other types.
3) If an item uses complex asset values rather than cash-flow determinants as scenario tree variables, it is helpful if there is a good explanation as to why this is done.
4) It is helpful if an item presents thorough tests of any scenario model used, or at least describes any potential limitations the model may have.
5) Models with “analytical” solutions may be useful on occasion to illustrate general qualitative points. However, such models typically leave out important details about any real situations.
6) If an item uses models with cash-flows and decisions that occur in continuous time, it is useful if there is a good explanation of why this is important. Such models usually require the analyst to make approximations in other areas.
7) It is helpful if any use of mathematical language (e.g., partial differential equations or integrals, or the mathematical aspects of any econometric analysis) is supportive of the natural language description of the issues involved, and does not replace it or overwhelm it.
8) If an item contains a state pricing analysis is formulated in mathematical terms using partial differential equations, it is helpful if it makes it clear that this is a way of doing a sum over states in that state pricing exercise.
9) Terms like “risk-neutral valuation” or “risk-neutral probabilities” tend to be confusing, and obscure the ideas upon which in MAP analyses are based. Alternatives like “risk-adjusted probabilities” are clearer.
10) Generally speaking, it is better if the numerical analysis methods used to find results in a MAP analysis are as efficient as possible, so as not to restrict the scope of the modelling. More specifically, the justification for the use of discrete recombining trees to organise the numerical integration in a continuous state pricing analysis, or to manage a dynamic programming analysis, is sometimes made on the basis that these trees look like scenario trees themselves. This is not a good reason to use these methods.

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